EXERCISES

Q.1 (3)

$v = r\omega \Rightarrow \omega = \frac{v}{r}$ = constant [As v and r are constant]

Q.2 (1)
In uniform circular motion (constant angular velocity) kinetic energy remains constant but due to change in velocity of particle its momentum varies.

Q.3 (3)

$\omega_{min} = \frac{2\pi}{60}\frac{Rad}{min}$ and $\omega_{hr} = \frac{2\pi}{12\times 60}\frac{Rad}{min}$

$\therefore \frac{\omega_{min}}{\omega_{hr}} = \frac{2\pi/60}{2\pi/12\times 60}$

Q.4 (4)

$120\text{ rev/min} = 120\times\frac{2\pi}{60}\text{rad/sec} = 4\pi\text{ rad/sec}$

Q.5 (1)

$\omega = \frac{v}{r} = \frac{100}{100} = 1\text{rad/s}$

Q.6 (2)

$\vec{v} = \vec{\omega}\times\vec{r} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -4 & 1\\ 5 & -6 & 6\end{vmatrix} = -18\hat{i} - 13\hat{j} + 2\hat{k}$.

Q.7 (3)

Q.8 (3)

Centripetal acceleration $= \frac{v^2}{r}$ = constant. Direction keeps changing.

Q.9 (1)

$\frac{a_R}{a_r} = \frac{\omega_R^2\times R}{\omega_r^2\times r} = \frac{T_r^2}{T_R^2}\times\frac{R}{r} = \frac{R}{r}$ [As $T_r = T_R$]

Q.10 (3)

Q.11 (4)
Centripetal force is constant in magnitude that means speed is constant and due to change in direction velocity is variable.

Q.12 (1)
Force is perpendicular to $\vec{v}$

$R = \frac{v^2}{a_\perp}$

$\Rightarrow R = \frac{mv^2}{F}$

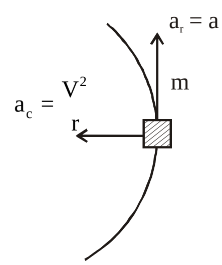
Q.13 (3)
$F_{C1} = F_{C2}$

$\Rightarrow \frac{mv_1^2}{r_1} = \frac{mv_2^2}{r_2}$

$\frac{v_1}{v_2} = \sqrt{\frac{r_1}{r_2}} = \frac{1}{\sqrt{2}}$

Q.14 (2)

$|\vec{a}| = \sqrt{a_c^2 + a_t^2} = \sqrt{\frac{v^2}{r^2} + a^2}$



Q.15 (4)

Q.16 (4)

$\frac{v^2}{rg} = \frac{h}{l}$

$\Rightarrow v = \sqrt{\frac{rgh}{l}} = \sqrt{\frac{50\times 1.5\times 9.8}{10}} = 8.57\text{ m/s}$

Q.17 (1)

Q.18 (4)

$F = mg - \frac{mv^2}{r}$.

Q.19 (3)
T = tension, W = weight and F = centrifugal force.

Q.20 (2)

$T = \frac{mv^2}{r}$

$$= \frac{0.5 \times (4)^2}{1} = 8N$$

Q.21 (1)

Q.22 (4)

$$v_{max} = \sqrt{\mu r g}$$

JEE-MAIN

OBJECTIVE QUESTIONS

Q.1 (3)

Speed $v_1 = \frac{2\pi r}{t}$

$$v_2 = \frac{2\pi r}{t}$$

$$\omega_1 = \frac{v_1}{r} = \frac{2\pi}{t} \Rightarrow \omega_2 = \frac{v_2}{2r} = \frac{2\pi}{t}$$

$$\omega_1 = \omega_2 \Rightarrow \frac{\omega_1}{\omega_2} = \frac{1}{1}$$

Q.2 (3)

$r = \frac{20}{\pi}$ m, a_t = constant

n = 2nd revolution

v = 80 m/s

$$\omega_0 = 0,\ \omega_f = \frac{v}{r} = \frac{80}{20/\pi} = 4\pi \text{ rad/sec}$$

$\theta = 2\pi \times 2 = 4\pi$

from 3rd equation

$\omega^2 = \omega_0^2 + 2\alpha\theta$

$\Rightarrow$ $(4\pi)^2 = 0^2 + 2 \times \alpha \times (4\pi)$

$\alpha = 2\pi$ rad/s^2

$$a_t = \alpha r = 2\pi \times \frac{20}{\pi} = 40 \text{ m/s}^2$$

Q.3 (4)

Speed = constant

In uniform circular motion, velocity and acceleration are constant in magnitude but direction is changes. Therefore velocity and acceleration both change.

Q.4 (4)

$$\omega_{second} = \frac{2\pi}{T} = \frac{2\pi}{60} \text{ rad/sec.}$$

$$v = \omega.r = \frac{2\pi}{60} \times 0.06 \text{ m/s} = 2\pi \text{ mm/s}$$

$$\Delta\vec{v} = \vec{v}_f - \vec{v}_i = \sqrt{2}\, v = 2\sqrt{2}\,\pi \text{ mm/s}$$

Q.5 (3)

Given, $\omega_0 = 0$, t = 2 sec.

$\theta = 0$, next 2 sec., $\theta = O_2$

$$\theta_1 = \frac{1}{2}\alpha t^2 = \frac{1}{2}\alpha 2^2 = 2\alpha$$

$$\theta_2 = \frac{1}{2}\alpha(2+2)^2 - \frac{1}{2}\alpha 2^2 = 6\alpha$$

$$\frac{\theta_2}{\theta_1} = \frac{6\alpha}{2\alpha} = 3$$

Q.6 (3)

$$\omega_1 = \frac{2\pi}{T_1},\quad \omega_2 = \frac{2\pi}{T_2}$$

$\omega_1 : \omega_2 = T_2 : T_1$

$T_1 = 12 \times 60 \times 60$ sec.

$T_2 = 60$ sec.

$\omega_1 : \omega_2 = 60 : (12 \times 60 \times 60)$

$\omega_1 : \omega_2 = 1 : 720$

Q.7 (1)

$$\omega = \frac{2\pi}{t}$$

where $t = 1\text{Day} = 24 \times 60 \times 60$ second

because earth complete one revolution is 24 hours about its own axis

$$w = \left(\frac{2\pi}{60 \times 60 \times 24}\right) \text{rad/s}$$

Q.8 (4)

Given

$a = 10\text{m/sec}^2 \Rightarrow \alpha = 5 \text{ rad/sec}^2$

$a = \alpha r$

$$r = \frac{10}{5} = 2 \text{ m}$$

Q.9 (1)

Given $\omega_0 = 0$, $\omega = 2\pi n = 2\pi \times \frac{210}{60}\frac{\text{rad}}{\text{sec}}$

from t = 5

$\omega = \omega_0 + \alpha t$

$$2\pi \times \frac{210}{60} = 0 + \alpha \times 5 \Rightarrow \alpha = 1.4\pi \frac{\text{rad}}{\text{sec}^2}$$

Q.10 (3)

$a_c = \frac{v^2}{r}$, radius is constant in case (a) and increase in case (b). So that magnitude of acceleration is constant in case (a) and decrease in case (b).

Q.11 (2)

$$a_c = \omega^2 R = \frac{4\pi^2}{T^2} R = \frac{4\times 3.14^2 \times 6400 \times 10^5}{(24\times 60\times 60)^2}$$

$$\omega^2 R = \frac{4\pi^2}{T^2} R = \frac{4\times 3.14^2 \times 6400 \times 10^5}{(24\times 60\times 60)^2} = 3.4 \text{cm/sec}^2$$

Q.12 (3)

Given r = 25 cm, n = 2

$\omega = 2\pi \times 2 \text{ rad/s} \Rightarrow a_c = \omega^2 r$

$= (4\pi)^2 \times 0.25 = 16\pi^2 \times 0.25 = 4\pi^2$

Q.13 (1)

Slope should be decreasing

$\alpha = \frac{d\omega}{dt} = \tan\theta$, if $\theta\downarrow$, $\alpha\downarrow$

Q.14 (3)

Given

$\omega = \theta^2 + 2\theta$

$$\frac{d\omega}{d\theta} = 2\theta + 2 \Rightarrow \left.\frac{d\omega}{d\theta}\right|_{t=1} = 2\theta + 2 = 4$$

$$\alpha = \frac{\omega d\omega}{d\theta} = (\theta^2 + 2\theta).(2\theta + 2) = 12 \text{ rad/sec}^2$$

Q.15 (2)

We know that

$v \le \sqrt{\mu r g}$

$v \le \sqrt{0.64 \times 20 \times 9.8}$

$v \le 11.2 \text{ m/s}$

Q.16 (4)

r = 144 m, m = 16 kg, T_{max} = 16 N

$T = \frac{mv^2}{r}$

$v = \sqrt{\frac{Tr}{M}} = \sqrt{\frac{16\times 144}{16}} = 12$ m/s

Q.17 (4)

$T = m\omega^2 r$

$\Rightarrow T^1 = 2T = m\omega_1^2 r$

$\omega_1 = \sqrt{2}\ \omega = \sqrt{2} \times 5 = \sqrt{50} \sim 7$ rev/min

Q.18 (1)

Uniformly rotating turn table means angular velocity is constant. New radius is half of the original value.

$r' = 2r$ and ω = constant

$v' = \omega r' = 2\omega r = 2v = 20$ cm/s

$a' = \omega^2 r' = 2\omega^2 r = 2a = 20$ cm/s^2

Q.19 (3)

For just slip $\Rightarrow \mu mg = m\omega^2 r$

here ω is double then radius is 1/4th

$r' = 4$ cm

Q.20 (2)

We know the Tension provides necessary centripetal force

So $T = m\omega^2 \ell$

Given m = 0.1, $\omega = 2\pi \times \frac{19}{\pi}$

$\ell = 1 \Rightarrow T = m\omega^2\ell$

$$T = 0.1 \times \left(2\pi \times \frac{10}{\pi}\right)^2 \times 1$$

$$= 0.1 \times 4\pi^2 \times \frac{100}{\pi^2} \times 1 = 40 \text{ N}$$

Q.21 (3)

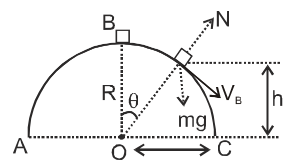
At t = 0,

$a_\perp = g\cos\theta$,

$$R = \frac{v^2}{a_\perp} = \frac{u^2}{g\cos\theta}$$

Q.22 (2)

Let the car looses the contact at angle θ with vertical



$$mg\cos\theta - N = \frac{mv^2}{R} \Rightarrow N = mg\cos\theta - \frac{mv^2}{R}$$

During descending on overbridge θ is incerese. So cos θ is decrease therefore normal reaction is decrease.

Q.23 (4)

For circular motion in vertical plane normal reaction is minimum at highest point and it is zero, minimum speed of motorbike is -

$$mg = \frac{mv^2}{R} \Rightarrow v = \sqrt{gR}$$

Q.24 (1)

$T - mg = \frac{mv^2}{r}$ (centripetal force at lowest point)

$$T = \frac{mv^2}{r} + mg$$

Q.25 (2)

For normal reaction at points A and B.

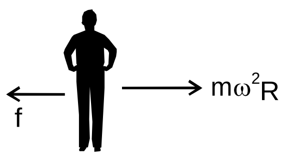
$$mg - N = \frac{mv^2}{r}$$

$$N = mg - \frac{mv^2}{r}$$

$\Rightarrow N_A > N_B$ and normal reaction at C is N_C = mg, so $N_C > N_A > N_B$

Q.26 (3)
Car will not slip when moving with speed v

Q.27 (1)



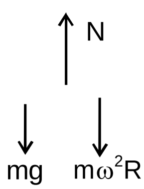
$$\mu mg \geq \frac{mv^2}{R}$$

$0.5\ mg \geq m \times (5)^2 \times R$

$$\frac{0.5 \times 10}{25} \geq R$$

$R \leq 0.2$ m

Q.28 (3)



Given
R = 10 m

m = 500 kg
$N = m\omega^2 R + mg$

$$= \frac{mv^2}{R} + mg = \frac{500 \times 400}{10} + 500 \times 10$$

= 25 kN

Q.29 (3)

$v = \sqrt{Rg \tan\theta}$

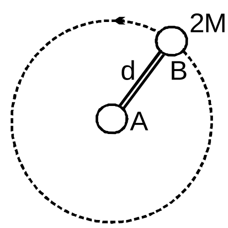
$R = 10\sqrt{3}$ m, $\theta = 30°$

$$= \sqrt{10\sqrt{3} \times 10 \times \frac{1}{\sqrt{3}}} = 10 \text{ m/sec} = 36 \text{ km/hr}$$

Q.30 (2)
Here required centripetal force is provided by friction force. Due to lack of sufficient centripetal force car thrown out of the road in taking a turn.

Q.31 (4)
In uniform circular motion
Force is towards centre

Q.32 (2)
Given



$$P = \frac{2\pi}{\omega} \Rightarrow \omega^{-1} = \frac{P}{2\pi}$$

$$T = 2M\,\omega^2 d = \frac{8\pi^2 Md}{P^2}$$

Q.33 (1)
The maximum bearable Tension

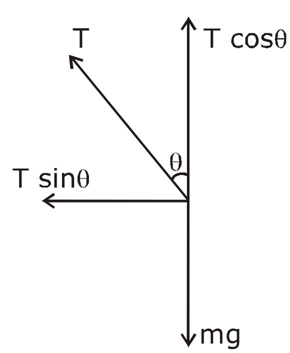
$$T = \frac{mv^2}{l}$$

T_{max} = 10 N,
m = 1, v = ?, l = 1

$$\upsilon = \sqrt{\frac{Tl}{m}} = \sqrt{\frac{100 \times 1}{1}} = 10 \text{ m/s}$$

Q.34 (3)
At highest point velocity is zero.
After word it fall freely.

Q.35 (1)



$r = \ell \sin\theta$
$T \sin\theta = mw^2\ell$
$T \cos\theta = mg$

Q.36 (3)

Given that

v = 72 km/h., r = 80 m

We know that

$$\tan\theta = \frac{v^2}{rg} = \frac{20\times 20}{80\times 10} = \frac{1}{2}$$

$$\theta = \tan^{-1}\left(\frac{1}{2}\right)$$

Q.37 (3)

We know that

$\upsilon^2 = rg\tan\theta$ (θ is same) $\Rightarrow \upsilon^2 = rg$

Case 1

$r_1 = 20$ m, $v_1 = v$

$r_2 = r$, $v_2 = 1.1v$

$$\frac{v_2^2}{v_1^2} = \frac{r_2 g}{r_1 g} \Rightarrow \frac{(1.1v)^2}{v^2} = \frac{r_2}{r_1}$$

$$1.21 = \frac{r}{20} \Rightarrow r = 24.2\,m$$

JEE-ADVANCED

OBJECTIVE QUESTIONS

Q.1 (C)

$\theta_A \propto t^2$ $\quad$ $\theta_B \propto t$

$\theta_A = k_1 t^2$ $\quad$ $\theta_B = k_2 t$

From given conditon calculate k_1 and k_2

$2\pi = k_1 \times \pi$ $\quad$ $\pi = k_2 \times 4\pi$

$k_1 = 2$ $\quad$ $k_2 = 1/4$

$\theta_A = 2t^2$ $\quad$ $\theta_B = t/4$

$$w_A = \frac{d\theta_A}{dt} = 4t \qquad w_B = \frac{d\theta_B}{dt} = \frac{1}{4}$$

$$\left(\frac{d\theta_A}{dt}\right)_{t=5sec} = 20 \qquad \left(\frac{d\theta_B}{dt}\right)_{t=5sec} = \frac{1}{4}$$

$\omega_A : \omega_B = 80 : 1$

Q.2 (D)

$\omega_{QP} = 2\pi - 5\pi = -3\pi$ rad/s

$\omega_{RP} = 3\pi - 5\pi = -2\pi$ rad/s

Time when Q particle reaches at P $= t_1 = \frac{\pi/2}{3\pi} = \frac{1}{6}$ sec.

$$t_2 = \frac{5\pi/2}{3\pi} = \frac{5}{6} \text{ sec.}$$

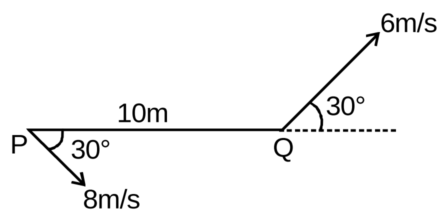
$$t_3 = \frac{9\pi/2}{3\pi} = \frac{3}{2} \text{ sec.}$$

Time where R particle reaches at P. $t_1 = \frac{\pi}{2\pi} = \frac{1}{2}$ sec.

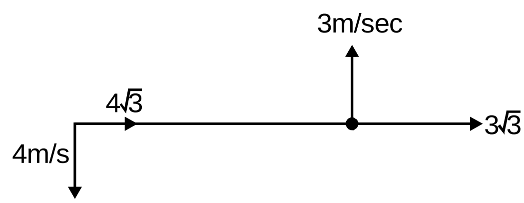
$$t_2 = \frac{3\pi}{2\pi} = \frac{3}{2} \text{ sec.}$$

Common time to reaches at P is $\frac{3}{2}$ sec.

Q.3 (D)



w.r.t to P



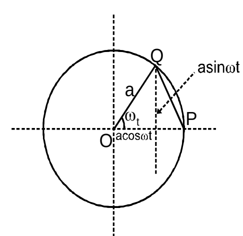
$$v_{\perp rel} = 8\sin 30° + 6\sin 30° = 7 \text{ m/s}$$

$$\omega = \frac{v_{\perp rel}}{R} = \frac{7}{10} = 0.7\,\text{rad/sec}$$

Q.4 (D)

$$PQ = \sqrt{(a - a\cos\omega t)^2 + (a\sin\omega t)^2}$$

$$= 2a\sin\left(\frac{\omega t}{2}\right)$$



Q.5 **(i) (A), (ii) (A)**

(i) At any moment $a_t = a_r$

$$a_t = -\frac{v^2}{R}$$

$$v\frac{dv}{ds} = -\frac{v^2}{R} \Rightarrow \frac{dv}{v} = -\frac{1}{R}ds$$

After integration $\log v = -\frac{S}{R} + C$ ...(i)

at t = 0, s = 0, $v = v_0$

$C = \log v_0$

from eq. (1) $\log\left(\frac{v}{v_0}\right) = -\frac{S}{R}$

$v = v_0 e^{-S/R}$

(ii) At any moment $a_t = a_v$ $a = \sqrt{2}\,a_r = \sqrt{2}.\frac{v^2}{R}$

Q.6 (A)

It can be observed that component of acceleration perpendicular to velocity is

$a_c = 4\ m/s^2$

$\therefore$ radius $= \frac{v^2}{a_c} = \frac{(2)^2}{4} = 1$ m.

Q.7 (B)

$F_C = mk^2 rt^2$

$a_C = k^2rt^2 = \frac{v^2}{r}$

$\Rightarrow v = krt$

$a_t = \frac{dv}{dt} = kr$

$F_t = mkr$

$\Rightarrow P = \vec{F}\cdot\vec{v}$ $(\because \vec{F}_C\cdot\vec{v} = 0)$

$P = \vec{F}_t\cdot\vec{v} = mkr \times krt$

$= mk^2r^2t$

Q.8 (B)

$K = \frac{1}{2}mv^2 = as^2 \Rightarrow v^2 = \frac{2as^2}{m}$

$a_C = \frac{v^2}{R} = \frac{2as^2}{mR}$

$a_t = v\frac{dv}{ds} = \frac{2as}{m}$

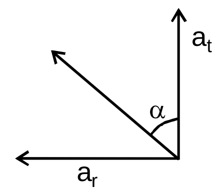
$$a = \sqrt{\left(\frac{2as^2}{mR}\right)^2 + \left(\frac{2as}{m}\right)^2} = \frac{2as}{m}\left(1+\frac{s^2}{R^2}\right)^{1/2}$$

Total force $= ma = 2as\left(1+\frac{s^2}{R^2}\right)^{1/2}$

Q.9 **(B)**

Given $v = a\sqrt{s}$

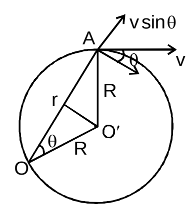
$a_t = \frac{vdv}{ds} = a\sqrt{s}.\frac{a}{2\sqrt{s}} = \frac{a^2}{2}$



$a_r = \frac{v^2}{R} = \frac{a^2 s}{R}$

$\tan\alpha = \frac{a_r}{a_t} = \frac{2s}{R}$

Q.10 **(D)**



$\frac{r}{2} = R\cos\theta$

$r = 2R\cos\theta$

After differentiable

$\frac{dr}{dt} = -2R\sin\theta\frac{d\theta}{dt} \Rightarrow \frac{dr}{dt} = v_{rad} = v\sin\theta$

$\frac{d\theta}{dt} = \omega$ (– ve because θ decreasing)

$v\sin\theta = 2R\sin\theta\omega$

$v = 2R\omega = 0.4$ m/s

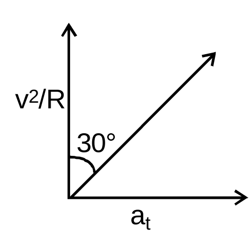
$a = \sqrt{a_t^2 + a_r^2}$ $\therefore$ ω = constant

$\Rightarrow a = a_r = \frac{v^2}{R}$

$\Rightarrow a_t = 0$

$\Rightarrow a_r = \frac{V^2}{R} = 32\ m/s^2$

Q.11 **(C)**



$a_t = \sqrt{3}\,t$

$\int dV = \int \sqrt{3}tdt$

$v = \frac{\sqrt{3}t^2}{2}$

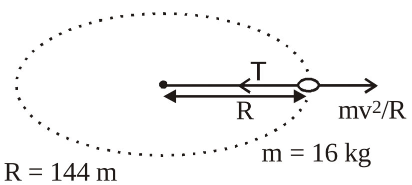
$\tan 30° = \frac{\sqrt{3}t.R}{\left(\frac{\sqrt{3}t^2}{2}\right)^2} \Rightarrow \frac{1}{\sqrt{3}} = \frac{4t}{\sqrt{3}t^4}$

$\Rightarrow\ t^4 = 4t \Rightarrow\ t^3 = (2)^2$

$\Rightarrow\ t = 2^{2/3}$ sec

Q.12 (D)

Given



$\frac{MV^2}{R} = T$

$T_{max} = 16$ N

$v_{max} = \sqrt{\frac{RT}{m}} \Rightarrow v_{max} = \sqrt{\frac{16\times144}{16}} = 12\,m/s$

Q.13 (A)

$v = r\omega$

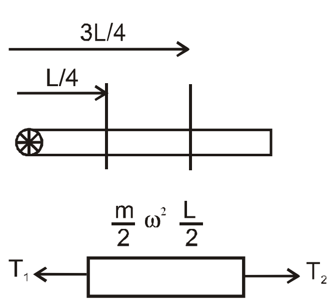
If $r \to r/2$

$\therefore\ v' = \frac{v}{2} = \frac{20}{2} = 10$ cm/sec

Turn table rotating uniformly $a_t = 0$

$a_r = \frac{v^2}{R}$; $a'_r = \frac{v'^2}{R/2} = \frac{20}{2} = 10$ cm/s²

Q.14 (A)



$T_1 - T_2 = \frac{M}{2}\omega^2\frac{L}{2}$

$T_1 > T_2$

Q.15 (C)

For water does not fall at topmost point of path that means at topmost point N should be greater than or equal to zero.

for N = 0, $mg = \frac{mv^2}{r}$

and for N > 0, $mg < \frac{mv^2}{r}$

so that mg is not greater than $\frac{mv^2}{r}$

Q.16 (A)

When train A moves form east to west

$mg - N_1 = \frac{m(v+\omega R)^2}{R}$

$\Rightarrow\ N_1 = mg - \frac{m(v+\omega R)^2}{R}$

$N_1 = F_1$

When train B moves from west to east

$mg - N_2 = \frac{m(v-\omega R)^2}{R} \Rightarrow N_2 = mg - \frac{m(v-\omega R)^2}{R}$

$N_2 = F_2$

$F_1 > F_2$

Q.17 (A)

$mg = m\omega^2 R$, $\omega = \sqrt{\frac{g}{R}}$

Q.18 (D)

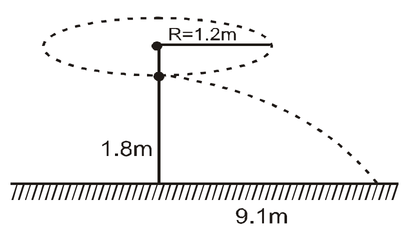
v = 72 km = 20m/s, r = 20m, g = 10 m/s²

To avoid skiding θ must be greater than

$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) = \tan{-1}\left(\frac{20\times20}{20\times10}\right)$

$\theta = \tan^{-1}(4)$

Q.19 (C)



The time taken to fall on ground = $\sqrt{\frac{2\times1.8}{9.8}} = \sqrt{\frac{36}{98}}$

velocity at time of string breaks

$v = \frac{\text{distance}}{\text{time}} \Rightarrow v = 9.1\sqrt{\frac{98}{36}}$

Centripetal acceleration = $\frac{v^2}{R} = \frac{9.1\times9.1\times98}{1.2\times36}$

= 187.856 = 188 m/s^2

Q.20 (D)

For M to be stationary

$T = Mg$ .... (1)

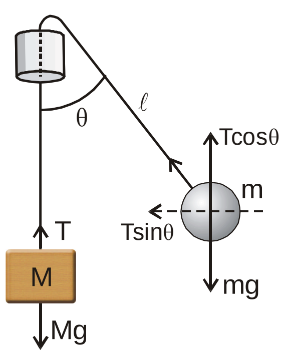
Also for mass m,

$T\cos\theta = mg$ .... (2)

$T\sin\theta = \frac{mv^2}{\ell\sin\theta}$ .... (3)

dividing (3) by (2)

$\tan\theta = \frac{v^2}{g\ell\sin\theta}$



$\Rightarrow v = \sqrt{\frac{g\ell}{\cos\theta}}.\sin\theta$

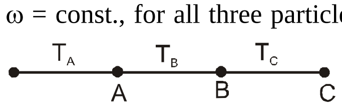
Time period $= \frac{2\pi R}{v} = \frac{2\pi\ell\sin\theta}{\sqrt{\frac{g\ell}{\cos\theta}}.\sin\theta}$

From (1) and (2) $\cos\theta = \frac{m}{M}$

then time period = $2\pi\sqrt{\frac{\ell m}{gM}}$

Q.21 (D)

ω = const., for all three particles



$\omega = \frac{v}{3\ell}$

$T_C = m\omega^2 3\ell$

$T_B - T_C = m\omega^2 2\ell$

$T_B = 5\,m\omega^2\ell$

$T_A - T_B = m\omega^2\ell$

$T_A = 6\,m\omega^2\ell$

$T_C : T_B : T_A :: 3 : 5 : 6$

Q.22 (B)

$F = kx,\ T_1 = ka = m\omega^2 2a$

$\Rightarrow \omega = \sqrt{\frac{k}{2m}}$

Time period $= \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{2m}{k}} = T$

$T_2 = 2ka = m\omega^2 3a$

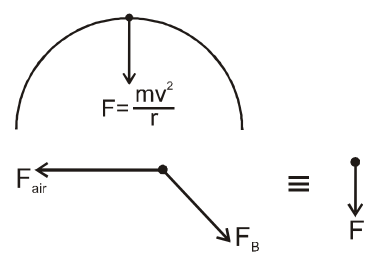
$\Rightarrow \omega = \sqrt{\frac{2k}{3m}}$

Time period = $2\pi\sqrt{\frac{3m}{2k}} = T'$

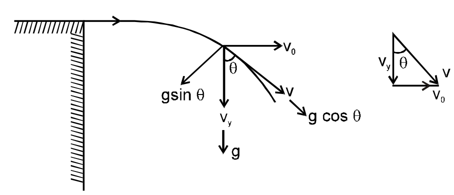
$T' = \left(\frac{\sqrt{3}}{2}\right)T$

Q.23 **(B)**

In uniform circular motion resultant horizontal force on the car must be towards the centre of circular path.



Q.24 **(A)**



As we know :

$a_C = \frac{v^2}{R}$ (centripetal acceleration)

From figure : $g\sin\theta = \frac{v^2}{R}$

$\Rightarrow g \cdot \frac{v_0}{v} = \frac{v^2}{R}$ (since $\sin\theta_i = \frac{v_0}{v}$)

$\Rightarrow R \propto v^3$

Q.25 (A)

Maximum retardation $a = \mu g$

For apply brakes sharply minimum distance require to stop.

$0 = v^2 - 2\mu g s$

$$\Rightarrow \quad s = \frac{v^2}{2\mu g}$$

For taking turn minimum radius is

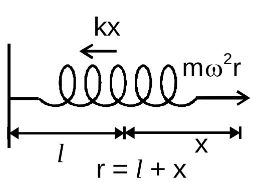
$$\mu g = \frac{v^2}{r},$$

$\Rightarrow r = \frac{v^2}{\mu g}$, here r is twice of s

so apply brakes sharply is safe for driver.

Q.26 (B)

$kx = m\omega^2 r$



$kx = m\omega^2 (l + x)$

$$x = \frac{m\omega^2\ell}{k - m\omega^2}$$

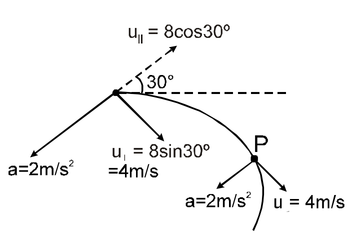
Q.27 (C)

The acceleration vector shall change the component of velocity $u_{\parallel}$ along the acceleration vector.

$$r = \frac{v^2}{a_n}$$

Radius of curvature r_{min} means v is minimum and a_n is maximum. This is at point P when component of velocity parallel to acceleration vector becomes zero, that is $u_{\parallel} = 0$.

$u_{\parallel} = 0$

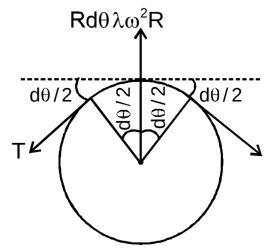


$$\therefore \quad R = \frac{u_\perp^2}{a} = \frac{4^2}{2} = 8 \text{ meter.}$$

Q.28 (C)

$$2T\sin\frac{d\theta}{2} = Rd\theta\lambda\omega^2R$$

If $d\theta$ is small

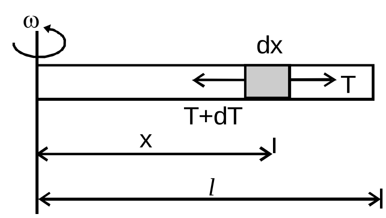


$$\sin\frac{d\theta}{2} \simeq \frac{d\theta}{2}$$

$$2T\frac{d\theta}{2} = Rd\theta\lambda\omega^2R$$

$$T = \lambda\omega^2R^2$$

Q.29 **(D)**



$$(T+dT) - T = \frac{m}{\ell} w^2 x\, dx$$

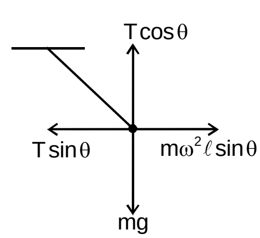
$$dT = \frac{m}{\ell}.\omega^2 x dx$$

Integrate with limit x to ℓ

$$T = \int_x^{\ell} \frac{m}{\ell}\omega^2 x dx$$

$$T = \frac{m\omega^2}{\ell}\left[\frac{x^2}{2}\right]_x^{\ell} \qquad = \frac{1}{2}\frac{m\omega^2}{\ell}[\ell^2 - x^2]$$

Q.30 **(B)**



T for simple pendulum = $2\pi\sqrt{\frac{\ell}{g}}$

For conical pendulum

$T \sin\theta = m\omega^2 l \sin\theta$

$\Rightarrow T = m\omega^2 l$

and $T \cos\theta = mg$

$$\Rightarrow \quad T = \frac{mg}{\cos\theta}$$

$$\text{Now,} \quad \frac{g}{\cos\theta} = \omega^2 l$$

$$\Rightarrow \omega = \sqrt{\frac{g}{l\cos\theta}}$$

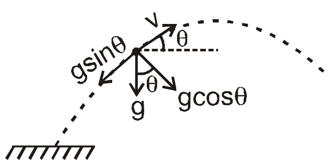
$$\therefore T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{\ell\cos\theta}{g}}$$

$$\therefore \frac{T_{\text{conical Pendulum}}}{T_{\text{simple Pendulum}}} = 2\pi\sqrt{\frac{\ell}{g}\cos\theta} \times \sqrt{\frac{g}{\ell}} \times \frac{1}{2\pi}$$

$$\text{Ratio} = \sqrt{\cos\theta}$$

Q.31 (B)

Tangential acceleration = a_t = $g\sin\theta$



Normal acceleration = a_n = $g\cos\theta$

$a_t = a_n$

$g\sin\theta = g\cos\theta \Rightarrow \theta = 45°$

$\Rightarrow v_y = v_x$

$u_y - gt = u_x$

$20 - (10)t = 10$

t = 1 sec.

During downward motion

$a_t = a_n$

$v_y = -v_x$

$20 - 10t = -10 \Rightarrow t = 3$ sec.

JEE-ADVANCED

MCQ/COMPREHENSION/COLUMN MATCHING

Q.1 (B,D)

(B) There are other forces on the particle

(D) The resultant of the other forces varies in magnitude as well as in direction.

Q.2 (A, C, D)

In curved path, may be circular or parabolic.

In circular path speed and magnitude of acceleration are constant.

In parabolic path acceleration is constant.

Q.3 (A,D)

(A) During a period of 1 year displacement is equal to zero, so that average velocity is equal to zero.

(B) During a period of one year distance travel is not equal to zero. So that average speed is not equal to zero.

(C) During a period of first 6 month of the year change in velocity not equal to zero. So that average acceleration is not equal to zero.

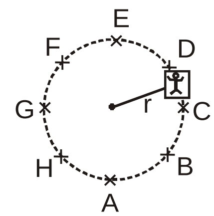
(D) In uniform circular motion instantaneous acceleration is act towards centre of circular path.

Q.4 (B,C,D,E)

$v = \sqrt{gr} \Rightarrow$ At A

$$N = mg + \frac{mv^2}{r} = 2mg \ [v = \sqrt{gr}]$$

at E



$$N + \frac{mv^2}{r} = mg$$

$\Rightarrow N = 0 \Rightarrow$ At G and C

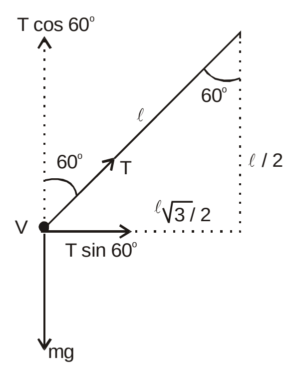
N = mg

Q.5 (B,C)

$$T - mg\cos\theta = \frac{mv^2}{L}$$

Tangential Acceleration = $g\sin\theta$

Q.6 (A,B,C,D)



$$\frac{T\sqrt{3}}{2} = \frac{mv^2}{(\ell\sqrt{3}/2)} \qquad ....(1)$$

$$\frac{T}{2} = mg \qquad ....(2)$$

Hence T = 2 mg , So (B) holds

From (1) & (2) $V^2 = 3g\ell/2$

$$\therefore \quad V = \sqrt{\frac{3\times 9.8\times 1.6}{2}}$$

$\therefore \quad V = 2.8\sqrt{3}$ m/s . So (C) hold

$$a_c = V^2/r = \frac{(3g\ell/2)}{(\ell\sqrt{3}/2)} = \sqrt{3} \times g = 9.8\sqrt{3} \text{ m/s}^2$$

$\therefore$ (D) holds

$$t = \frac{2\pi r}{v} = \frac{2\pi\sqrt{\ell\sqrt{3}/2}}{\sqrt{(3g\ell/2)}}$$

$t = 4\pi/7$ $\therefore$ (A) holds.

Q.7 (B,C)

$$a_t = \frac{dv}{dt} = a$$

$$\text{friction force on car} = m\sqrt{\left(\frac{v^2}{r}\right)^2 + a^2}$$

which is greater than $\frac{mv^2}{r}$

$$\mu_{min} = \frac{\sqrt{(v^2/r)^2 + a^2}}{g}$$

therefore it is not less than $\frac{a}{g}$ for safe turn.

Q.8 (B,C)
There is no friction between road and tyres of car so that car cannot remain in static equilibrium on curved section. Whenever speed of car is greater than or less than v car will slip.

Q.9 (B,D)
When speed of car is 36 km/hr, car can make a turn without skidding. If speed is less than 36 km/hr than tendency of slipping is downward so it will slip down. If speed is greater than 36 km/hr than tendency of slipping upward so it will slip up. If the car's turn at correct speed 36 km/hr

$$N \cos\theta = mg$$

$$N \sin\theta = \frac{mv^2}{r}$$

$$N = \sqrt{(mg)^2 + \left(\frac{mv^2}{r}\right)}$$

Q.10 (B)

Q.11 (A)

Q.12 (A)
(10 to 12)
The angular velocity and linear velocity are mutually perpendicular

$\therefore \ \vec{v}\cdot\vec{\omega} = 3x + 24 = 0$ or $x = -8$

The radius of circle $r = \frac{v}{\omega} = \frac{5}{10} = \frac{1}{2}$ meter

The acceleration of particle undergoing uniform circular motion is

$$\vec{a} = \vec{\omega}\times\vec{v} = (-8\hat{i} + 6\hat{j})\times(3\hat{i} + 4\hat{j}) = -50\hat{k}$$

$\therefore \ \vec{v}\cdot\vec{\omega} = 3x + 24 = 0$ or $x = -8$

Q.13 (D)

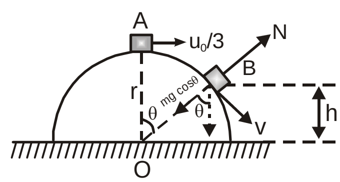
$$mg = \frac{mu_0^2}{r} \Rightarrow u_0 = \sqrt{gr}$$

Now, along vertical

$$r = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2r}{g}}$$

Along horizontal; $OP = 2u_0t = 2\sqrt{2}\ r$

Q.14 (B)
As at B it leaves the hemisphere,



$\therefore \quad N = 0$

$$mg \cos\theta = \frac{mV^2}{r}$$

$$mg\frac{h}{r} = \frac{mV^2}{r}$$

$mv^2 = mgh$ ....(1)

By energy conservation between A and B

$$mgr + \frac{1}{2}m\left(\frac{u_0}{3}\right)^2 = mgh + \frac{1}{2}mv^2$$

Put u_0 and mv^2 $\quad \therefore h = \frac{19r}{27}$

Q.15 (C)

As $a_c = \frac{v^2}{r} = g\cos\theta$

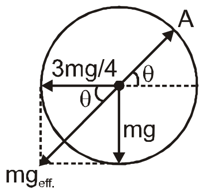
$\therefore \quad a_t = g \sin\theta$

$\therefore \quad a_{net} = g$

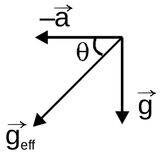
Alternate Solution :
when block leave only the force left is mg.

$\therefore \quad a_{net} = g.$

Q.16 (B)



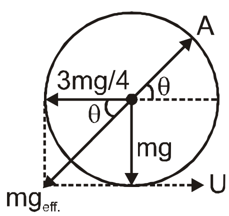
$\vec{g}_{eff} = \vec{g} - \vec{a}$



Tension would be minimum when it (tension) is along $\vec{g}_{eff}$

$\tan\theta = \dfrac{mg}{\frac{3}{4}mg} = \dfrac{4}{3} \therefore \theta = 53^\circ$.

Q.17 (C)



$V_{min} = \sqrt{\ell g_{eff}} = \sqrt{\ell \dfrac{5}{4} g} = \dfrac{\sqrt{5\ell g}}{2}$.

Q.18 (C)

$T_{max} = 6\ mg_{eff}\ \left(g_{eff} = \dfrac{5}{4} g\right)$

$= \dfrac{15}{2} mg$

Q.19 (A) q,s (B) p (C) p (D) q,r

From graph (a) $\Rightarrow \omega = k\theta$

where k is positive constant

angular acceleration $= \omega \dfrac{d\omega}{d\theta} = k\theta \times k = k^2\theta$

$\therefore$ angular acceleration is non uniform and directly proportional to θ.

$\therefore$ (A) q, s

From graph (b) $\Rightarrow \omega^2 = k\theta$.

Differentiating both sides with respect to θ.

$2\omega \dfrac{d\omega}{d\theta} = k$ or $\omega \dfrac{d\omega}{d\theta} = \dfrac{k}{2}$

Hence angular acceleration is uniform.

$\therefore$ (B) p

From graph (c)

$\Rightarrow \omega = kt$

angular acceleration $= \dfrac{d\omega}{dt} = k$

Hence angular acceleration is uniform

$\Rightarrow$ (C) p

From graph (d)

$\Rightarrow \omega = kt^2$

angular acceleration $= \dfrac{d\omega}{dt} = 2kt$

Hence angular acceleration is non uniform and directly proportional to t.

$\therefore$ (D) q,r

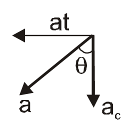
Q.20 (A) q (B) q, s (C) q, s (D) p, s

$v = 2t^2$

Tangential acceleration $a_t = 4t$

Centripetal acceleration $a_c = \dfrac{v^2}{R} = \dfrac{4t^4}{R}$

Angular speed $\omega = \dfrac{v}{R} = \dfrac{4t}{R}$,



$\tan\theta = \dfrac{a_t}{a_c} = \dfrac{4tR}{4t^4} = \dfrac{R}{t^3}$

NUMERICAL VALUE BASED

Q.1 10 m/s

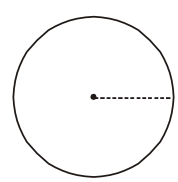
$R_C = \dfrac{U^2 \cos^2 37^\circ}{g}$

$\Rightarrow 6.4 = \dfrac{U^2 \times (4/5)^2}{10}$

Q.2 [0625]

$v = \omega R$

$v' = \omega(R - 5) = \dfrac{\omega R}{5}$



$5R - 25 = R$

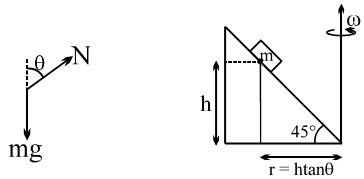
$R = \dfrac{25}{4} m = 6.25$ m

Q.3 [0050]

$R = 5m,\ h = 2m,\ \Delta x = 10m,\ v\sqrt{\dfrac{2h}{g}} = \Delta x,\ a_c = \dfrac{v^2}{R}$

Q.4 [2]

FBD of block in ground frame



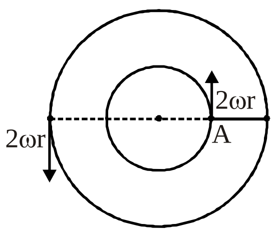
$N\cos\theta = mg$

$N\sin\theta = m\omega^2 r$ [centripetal force]

$\Rightarrow \tan\theta = \omega^2 r/g$

$\Rightarrow \omega = \sqrt{\frac{g\tan\theta}{r}} = \sqrt{\frac{g}{h}}(\tan\theta) = 2 \times 1 = 2$ rad/s

Q.5 [24]



Q.6 [128 sec.]

$$\theta = \frac{14 \times 10^8}{1.5 \times 10^{11}} = \frac{14}{1.5} \times 10^{-3}$$

$$\omega = \frac{2\pi}{24 \times 3600}$$

$$t = \frac{\theta}{\omega}$$

$$t = \frac{14}{1.5} \times 10^{-3} \times \frac{24 \times 3600}{2\pi}$$

$$t = \frac{14 \times 8 \times 3.6}{\pi} \simeq 128 \text{ sec.}$$

Q.7 [2]

Given $\frac{dp}{dt} = cv^n$

$\therefore \frac{mv^2}{r} = cv^n$

On comparing n = 2

Q.8 [0010]

The lift goes down with retardation means acceleration is upward, let it be a.

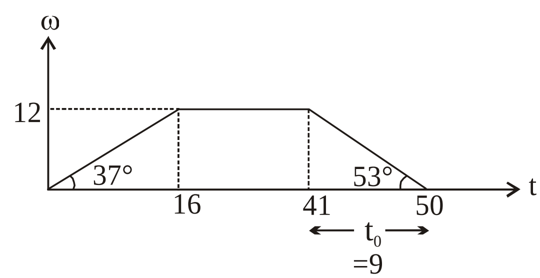
$$T = 2\pi\sqrt{\frac{h}{g_{eff}}} = 2\pi\sqrt{\frac{h}{g+a}}$$

$$\Rightarrow \quad 2 = 2\pi\sqrt{\frac{2}{10+a}} \Rightarrow a = 10$$

Q.9 [0009]

$$<\omega> = \frac{\int \omega\, dt}{\int dt} = \frac{\text{Area under graph}}{\text{time}}$$

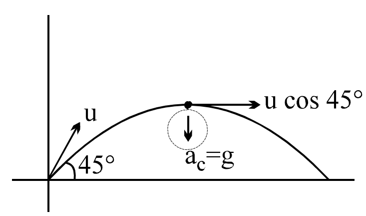
$$= \frac{\frac{1}{2} \times 12[25+50]}{50} = 9 \text{ r/s.}$$



Q.10 [20 m/s²]

At highest point $a_c = g$

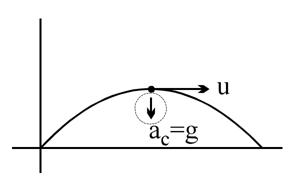
$$\frac{u^2\cos^2 45°}{R_c} = g \ [R_c = \text{Radius of curvature}]$$



$$\frac{u^2}{2R_c} = g \quad ....(1)$$

Now when be moves along the same path with constant speed u, then at top point, since radius of curvature (R_c) remains same

$$\frac{u^2}{R_c} = a_c \ ....(2)$$



from (1) and (2)

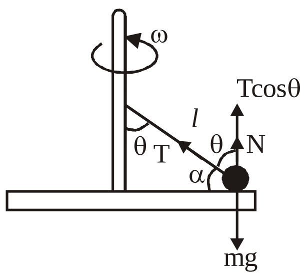
$$\frac{1}{2} = \frac{g}{a_c}$$

$\Rightarrow a_c = 2g$

$a_c = 20 \text{ m/s}^2$

Q.11 [5]

$T\sin\theta = m\omega^2\ell \sin\theta$



$T = ml\omega^2$

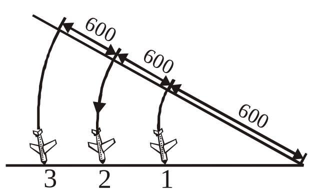
$T\cos\theta + N = mg$

$N = mg - ml\omega^2\cos\theta$

$$N = 1 \times 10 - 1 \times 0.1 \times \frac{1}{2} \times 100 = 5$$

Q.12 [50 m/s^2]

Angular velocity all aircraft will be same

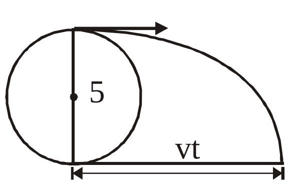


$$\omega = \frac{V_2}{R_2} = \frac{720 \times \frac{1000}{3600}}{1200} = \frac{1}{6} \text{ rad/s}$$

$$a_3 = \omega^2 R_3 = \left(\frac{1}{6}\right)^2 (1800) = 50 \text{ m/s}^2$$

Q.13 [5 m]

$$5 = \frac{1}{2} \times gt^2$$



t = 1 sec.

$$v = \frac{2\pi}{\pi} \times 2.5 = 5 \text{ m/s}$$

S = vt = 5 m

Q.14 [0005]

$T = mg$

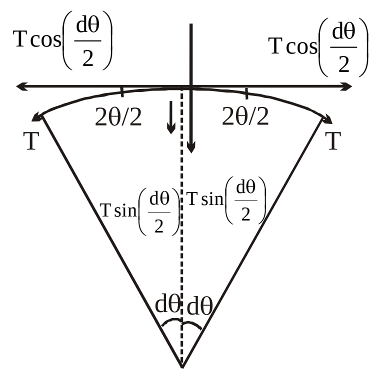
$m r_{max}\, \omega^2 = T + \mu mg$

$m r_{min}\, \omega^2 = T - \mu mg$

$m(r_{max} + r_{min})\omega^2 = 2mg$

$$\therefore\ r_{max} + r_{min} = \frac{2g}{\omega^2} = 5 \text{ m}$$

Q.15 $v = \sqrt{\dfrac{k(\Delta x)2\pi R}{m_0\ell}} = 1100$



$k\Delta x = T = k(2\pi r - \ell)$

$$2T\sin\left(\frac{d\theta}{2}\right) = (dm)\frac{v^2}{R}$$

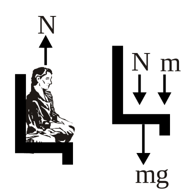
$$Td\theta = \left(\frac{m_0\ell}{2\pi r}\right) r \cdot (2d\theta) \cdot \frac{v^2}{r}$$

$$k(2\pi r - \ell) = \frac{m_0\ell v^2}{\pi r}$$

$$\therefore\ v^2 = \frac{k\pi r(2\pi r - \ell)}{m_0\ell}$$

Q.16 [0200]

$$N_b - mg = \frac{mv^2}{R}$$



$N_b = mg + \dfrac{mv^2}{R}$

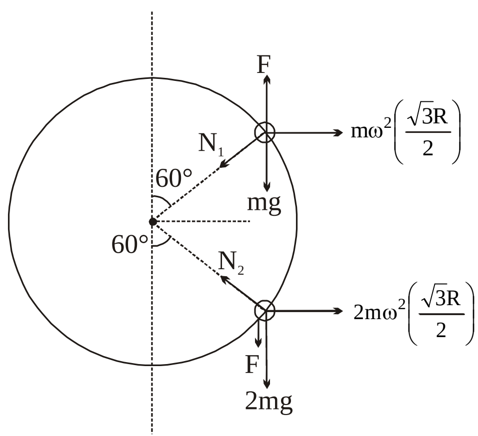
$N_T + mg = \dfrac{mv^2}{R}$

$N_T = \dfrac{mv^2}{R} - mg = \dfrac{1}{3}\left(mg + \dfrac{mv^2}{R}\right)$

$\dfrac{2}{3}\dfrac{mv^2}{R} = \dfrac{4}{3}mg$

$v = \sqrt{2gR} = \sqrt{20 \times 2000} = 200$ m/s

Q.17 [0002]



$F - N_1 \cos 60° = mg$ .... (1)

$N_1 \sin 60° = m\omega^2\left(\dfrac{\sqrt{3}R}{2}\right)$ ... (2)

$F - \dfrac{m\omega^2 R}{2} = mg$ .....(A)

$N_2 \cos 60° = F + 2mg$ ..... (3)

$N_2 \sin 60° = 2m\omega^2\left(\dfrac{\sqrt{3}R}{2}\right)$ .... (4)

$m\omega^2 R = F + 2mg$ ..... (B)

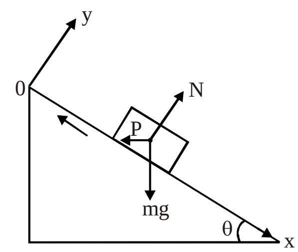
$\dfrac{1}{2} m\omega^2 R = 3mg \therefore \omega = \sqrt{\dfrac{6g}{R}}$

Q.18 [0003]
As shown in figure, the forces acting on the block are the gravitational force mg, the normal reaction N, the static friction f, and the cenrifugal

force with $f = \mu_s N$, $P = m\omega^2 r$. Thus the conditions for equilibrium are

$mg \sin\theta = P\cos\theta + \mu_s N$,

$N = mg\cos\theta + P\sin\theta$



Hence $mg\sin\theta = P\cos\theta + \mu_s mg\cos\theta + \mu_s P\sin\theta$,

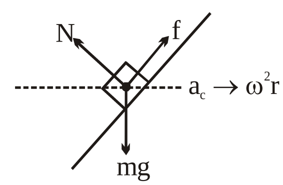
giving $P = \left(\dfrac{\sin\theta - \mu_s\cos\theta}{\cos\theta + \mu_s\sin\theta}\right) mg = m\omega^2 r$,

or $\omega^2 = \left(\dfrac{\sin\theta - \mu_s\cos\theta}{\cos\theta + \mu_s\sin\theta}\right) = \dfrac{g}{r}$

$= \left(\dfrac{\frac{3}{5} - \frac{1}{4}\cdot\frac{4}{5}}{\frac{4}{5} + \frac{1}{4}\cdot\frac{3}{5}}\right)\dfrac{9.8}{0.4} = 10.3$

$\omega = 3.2$ rad/s

Q.19 [0010]
$f\cos\theta - N\sin\theta = m\omega^2 r$
$f\sin\theta + N\cos\theta = mg$
for limiting condtion $f = \mu N$



$\Rightarrow \dfrac{\omega^2 r}{g} = \dfrac{\mu\cos\theta - \sin\theta}{\cos\theta + \mu\sin\theta} = \dfrac{0.2}{6.4}$

$\Rightarrow$ T = 10 s

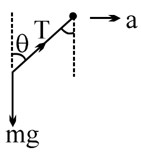
Q.20 [120]
In first time interval

$T\cos\theta = mg$

$T \sin\theta = mg$

$\Rightarrow \quad T = m\sqrt{g^2 + a^2}$

$g^2 + a^2 = \dfrac{25}{16} \times g^2 \Rightarrow a = \dfrac{3}{4} g$

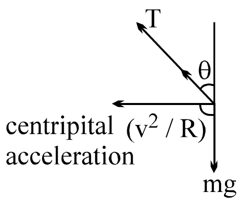


At t = 4 velocity v = at = 3g

In II[nd] time interval, string vertical means $\theta = 0$

$\Rightarrow \quad a = 0$

$\Rightarrow \quad T = mg$



In III[rd] time interval

v = 3g $\qquad T \sin\theta = \dfrac{mv^2}{R}$

$T \cos\theta = mg$

solving

$$T = m\sqrt{g^2 + \left(\frac{v^2}{R}\right)^2} = 5/4\ mg$$

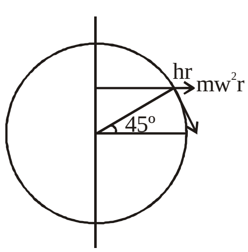
$$\Rightarrow \quad \left(\frac{v^2}{R}\right) = \frac{3}{4} g$$

$$\Rightarrow \quad R = \frac{9^2 g^2}{\frac{3}{4} g} = 12g = 120 \text{ m}$$

KVPY

PREVIOUS YEAR'S

Q.1 (C)

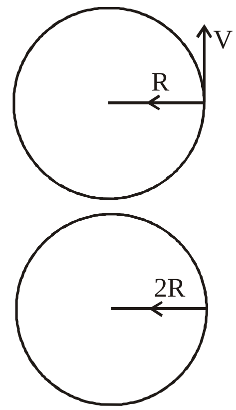


$F_r = m\omega^2 r \cos 45^\circ$

where $r = R\cos 45^\circ$

$$F_r = \frac{m\omega^2 R}{2}$$

Q.2 (C)



$$T = \frac{2\pi R}{V}$$

$$a_C = \frac{V^2}{R}$$

$$8a_C = \frac{V'^2}{2R}$$

$$(8)\frac{V^2}{R} = \frac{V'R}{2R}$$

$V'^2 = 16V^2$

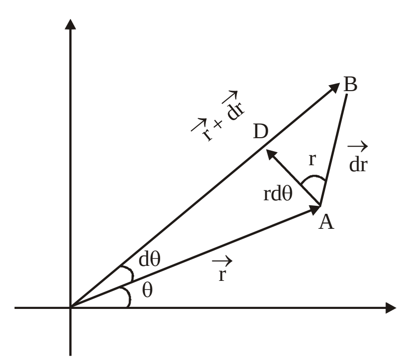
$V' = 4V$

$$\therefore \text{ Time period} = \frac{(2\pi)R'}{V'}$$

$$= \frac{(2\pi)2R}{4V}$$

$$= \frac{\pi R}{V} = (T/2)$$

Q.3 (B)



$|\overline{AB}|$ = Direction of resultant velocity

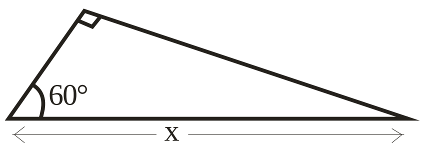
$\overline{AD}$ = Direction of tangential velocity

$$\forall \tan\alpha = \frac{dr}{rd\theta} = \frac{r}{r}$$

tan α = 1

α = 45º

Q.4 (A)



$$\tan 60° = \frac{L}{R} = \frac{1}{1/\sqrt{3}} = \sqrt{3} = \tan\theta$$

$$\therefore\ x = \sqrt{1+\frac{1}{3}} = \frac{2}{\sqrt{3}}$$

$$\cos\theta = \frac{R^2 + x^2 - L^2}{2Rx}$$

$\Rightarrow R^2 + x^2 - L^2 = 2Rx\cos\theta$

$$\Rightarrow 2x\frac{dx}{dt} = 2R\left[x(-\sin\theta) + \cos\theta\frac{dx}{dt}\right]$$

$$\frac{dx}{dt}[x - R\cos\theta] = -Rx\sin\theta\frac{d\theta}{dt}$$

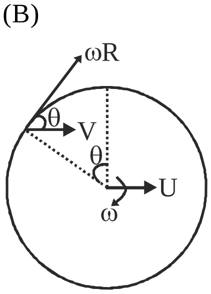
$$\therefore -\frac{dx}{dt} = v \text{ and } \frac{d\theta}{dt} = \omega$$

$$\Rightarrow v = \frac{Rx\sin\theta\omega}{x - R\cos\theta}$$

$$\frac{\frac{1}{\sqrt{3}}\frac{2}{\sqrt{3}}.\frac{\sqrt{3}}{2}.\omega}{\frac{2}{\sqrt{3}} - \frac{1}{\sqrt{3}}.\frac{1}{2}} = \frac{\frac{\omega}{\sqrt{3}}}{\frac{1}{\sqrt{3}}\frac{3}{2}} = \frac{2\omega}{3}$$

$$v = \frac{2\omega}{3}$$

Q.5 (B)



$$h_{max} = \frac{u^2\sin^2\theta}{2g} + R\cos\theta$$

$$\text{for } h_{max} \Rightarrow \frac{dh_{max}}{d\theta} = 0$$

$$\text{Solving we get } \frac{u^2}{2g} + \frac{gR^2}{2u^2}$$

JEE MAIN
PREVIOUS YEAR'S

Q.1 (2)

Particle is in uniform circular motion.

$$\text{Time Period T} = \frac{0.1s}{30º}\times 360º = 1.2s$$

Now,

$F = -kx = -m\omega^2 x$

$$\frac{F}{m} = -\omega^2 x = -\left(\frac{2\pi}{T}\right)^2 x = -\frac{4\pi^2}{T^2}x$$

$$= -\frac{4\times 9.87}{(1.2)^2}\times(-0.36) = +9.87\frac{N}{kg}$$

Q.2 (2)

$$F = \frac{C}{r^3} = m\omega^2 r$$

$$\therefore\ \omega^2\ \alpha\ \frac{1}{r^4}$$

$$\therefore\ \omega\ \alpha\ \frac{1}{r^2}$$

$\therefore\ T\ \alpha\ r^2$

Q.3 (4)

$N = m\omega^2 R$

$$N = m\left[\frac{4\pi^2}{T^2}\right]R \qquad .....(1)$$

Given m = 0.2 kg, T = 40 S, R = 0.2 m

Put values in equation (1)

$N = 9.859 \times 10^{-4}$ N

Q.4 (4)

Statement I :

$v_{max} = \sqrt{\mu Rg} = \sqrt{(0.2)\times 2\times 9.8}$

v_{max} = 1.97 m/s

7 km/h = 1.944 m/s

Speed is lower than v_{max}, hence it can take safe turn.

Statement II :

$$v_{max} = \sqrt{Rg\left[\frac{\tan\theta + \mu}{1 - \mu\tan\theta}\right]}$$

$$= \sqrt{2 \times 9.8\left[\frac{1+0.2}{1-0.2}\right]} = 5.42 \text{ m/s}$$

18.5 km/h = 5.14 m/s

Speed is lower than v_{max}, hence it can take safe turn.

Q.5 (2)

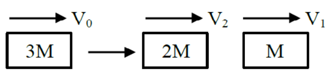
$$\mu_s N = \frac{mv^2}{R}$$

$$N = \frac{mv^2}{\mu_s R} = mg + F_L$$

$$F_L = \frac{mv^2}{\mu_s R} - mg$$

Q.6 (2)

Q.7 (3)



$3MV_0 = 2MV_2 + MV_1$

$3V_0 = 2V_2 + V_1$

$120 = 2V_2 + 60 \Rightarrow V_2 = 30$ m/s

$$\frac{\Delta K.E.}{K.E.} = \frac{\frac{1}{2}MV_1^2 + \frac{1}{2}2MV_2^2 - \frac{1}{2}3MV_0^2}{\frac{1}{2}3MV_0^2}$$

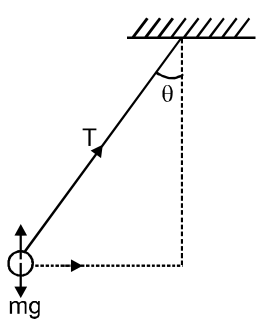
$$= \frac{V_1^2 + 2V_2^2 - 3V_0^2}{3V_0^2} = \frac{3600 + 1800 - 4800}{4800}$$

$$= \frac{1}{8}$$

Q.8 (2)

JEE-ADVACNED PREVIOUS YEAR'S

Q.1 (D)



$T \sin\theta = m L \sin\theta\, \omega^2$

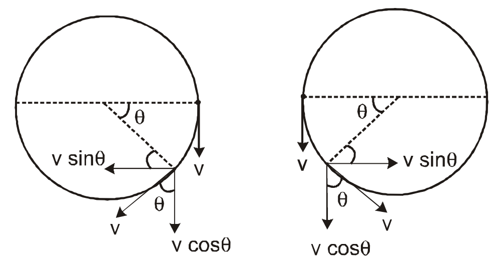
$324 = 0.5 \times 0.5 \times \omega^2$

$$\omega^2 = \frac{324}{0.5 \times 0.5}$$

$$\omega = \sqrt{\frac{324}{0.5 \times 0.5}}$$

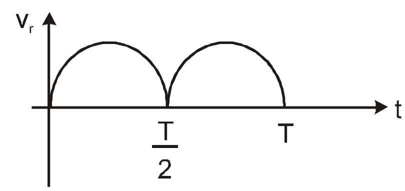
$$\omega = \frac{18}{0.5} = 36 \text{ rad/sec.}$$

Q.2 (A)



$v_r = |2 v \sin \theta)|$

$= |2v \sin \omega t)|$



Q.3 (B)

$$a = \omega^2 r$$

$$\Rightarrow \int_0^v v\,dv = \omega^2 \int_{R/2}^r r\,dr \quad \Rightarrow v = \omega\sqrt{r^2 - \frac{R^2}{4}}$$

$$\& \int_{R/2}^r \frac{dr}{\sqrt{r^2 - \frac{R^2}{4}}} = \omega \int_0^t dt$$

$$\Rightarrow r = \frac{R}{4}\left(e^{\omega t} + e^{-\omega t}\right)$$

Hence, (B)

Q.4 (D)

$$\vec{F}_{rot} = -m\omega^2 r\hat{i} + 2mv_{rot}\omega(-\hat{j}) + m\omega^2 r\hat{i}$$

$$= 2mv_{rot}\omega(-\hat{j})$$

$$= 2m\frac{\omega R}{4}\left(e^{\omega t} - e^{-\omega t}\right)\omega(-\hat{j})$$

$$\vec{F}_{net} = -\vec{F}_{rot} + mg\hat{k} = \frac{m\omega^2 R}{2}\left(e^{\omega t} - e^{-\omega t}\right)\hat{j} + mg\hat{k}$$

Hence, (D)

Work, Power and Energy

EXERCISES

ELEMENTARYS

Q.1 (3)
W = (force) (displacement) = (force) (zero) = 0

Q.2 (1)

Joule = (Newton) (Metre) = $\frac{4\,\text{Newton}}{4} \times \frac{4\,\text{Metre}}{4} = \frac{\text{Joule}}{16}$

Hence : 1 Joule = 16 joule (Joule is new unit of energy)

Q.3 (4)

Stopping distance $S \propto u^2$. If the speed is doubled then the stopping distance will be four times.

Q.4 (2)
Work done = Force × displacement
= Weight of the book × Height of the book shelf

Q.5 (4)

$$W = \vec{F}.\vec{s} = (5\hat{i} + 6\hat{j} - 4\hat{k}).(6\hat{i} + 5\hat{k}) = 30 - 20 = 10\,\text{units}$$

Q.6 (4)

$$s = \frac{t^2}{4} \therefore ds = \frac{t}{2}dt$$

$$F = ma = \frac{md^2s}{dt^2} = \frac{6d^2}{dt^2}\left[\frac{t^2}{4}\right] = 3\,N$$

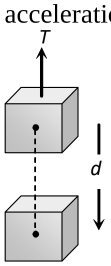
Now

$$W = \int_0^2 F\,ds = \int_0^2 3\frac{t}{2}dt = \frac{3}{2}\left[\frac{t^2}{2}\right]_0^2 = \frac{3}{4}\left[(2)^2 - (0)^2\right] = 3J$$

Q.7 (2)

$$W\int_0^{x_1} F.dx = \int_0^{x_1} Cx\,dx = C\left[\frac{x^2}{2}\right]_0^{x_1} = \frac{1}{2}Cx_1^2$$

Q.8 (3)
When the block moves vertically downward with acceleration $\frac{g}{4}$ then tension in the cord

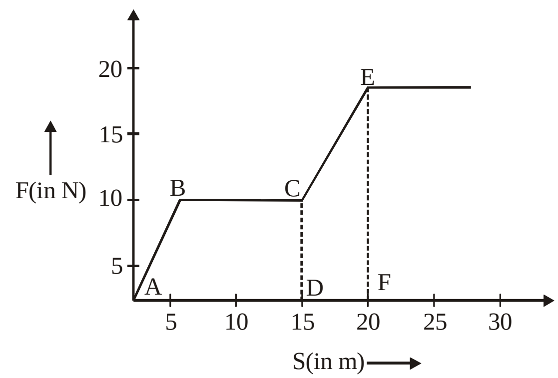


$$T = M\left(g - \frac{g}{4}\right) = \frac{3}{4}Mg$$

Work done by the cord = $\vec{F}.\vec{s} = Fs\cos\theta$

$$= Td\cos(180^\circ) = -\left(\frac{3\,Mg}{4}\right) \times d = -3Mg\frac{d}{4}$$

Q.9 (2)

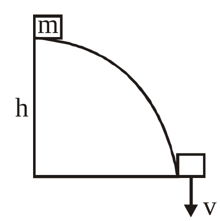


Work done, W = area under F-S graph from S = 0 m to x = 20 m
= Area of trapezium ABCD + Area of trapezium CEFD

$$= \frac{1}{2} \times (10 + 15) \times 10 + \frac{1}{2} \times (10 + 20) \times 5$$

= 125 + 75 = 200 J.

Q.10 (2)



Here, mass of the block, m = 3kg
Initial speed of the block, u = 0 (as it starts from rest)
Final speed of the block, v = 4 m/s
Height, h (in this case the radius of quarter circle) = 2m

$$\Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \frac{1}{2}mv^2 - 0$$

$$= \frac{1}{2}(3kg)(4m/s)^2 = 24J$$

The work done by the gravitational force is
W_g = mgh = (3 kg) (10 m/s^2) (2m) = 60 J
If W_f is the work done by the friction, then according to work energy theorem,

$W_g + W_f = \Delta K$

or $W_f = \Delta K - W_g = 24\text{ J} - 60\text{ J} = -36\text{ J}$

As work done against friction is equal and opposite to work done by the friction,

$\therefore$ The amount of work done against friction is 36 J.

Q.11 (4)

According to work-energy theorem, the work done by the net force on the body is equal to the change in its kinetic energy.

i.e., $W = K_f - K_i$.

Q.12 (3)

According to work-energy theorem

W = Change in kinetic energy

$$FS\cos\theta = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

Substituting the given values, we get

$20 \times 4 \times \cos\theta = 40 - 0$

$(\because u = 0)$

or $\cos\theta = \frac{40}{80} = \frac{1}{2}$ or $\theta = \cos^{-1}\left(\frac{1}{2}\right) = 60^o$

Q.13 (2)

(Applied force - frictional force) × distance = Gain in kinetic energy.

$\therefore$ $(20 - f) \times 2 = 10$ or $20 - f = 5$ or $f = 15$ N.

Q.14 (3)

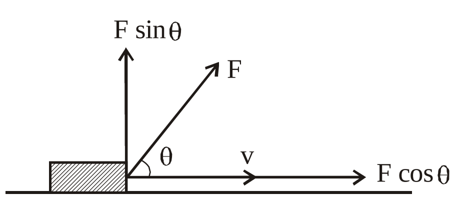
Power = $\frac{\text{Work}}{\text{Time}}$

$\therefore$ $P = \frac{mgh}{t}$ or $t = \frac{mgh}{P}$

Substituting the given values, we get

$t = \frac{200 \times 10 \times 40}{10 \times 10^3} = 8s$

Q.15 (1)



Power = (component of force in the direction of velocity)

$= F\cos\theta v$

Q.16 (4)

In compression or extension of a spring work is done against restoring force.

In moving a body against gravity work is done against gravitational force of attraction.

It means in all three cases potential energy of the system increases.

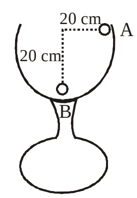
But when the bubble rises in the direction of upthrust force then system works so the potential energy of the system decreases.

Q.17 (2)

According to the conservation of energy, kinetic energy at A + potential energy at B

$$\Rightarrow 0 + mgh = \frac{1}{2}mv^2 + 0$$

or $v^2 = 2gh = 2 \times 9.8 \times 0.20$ ($\because$ h = radius = 20 cm = 0.2m)



According to work - energy theorem,

Work done on the ball = change in kinetic energy

$= \frac{1}{2}mv^2 - (0)^2$ $= \frac{1}{2} \times \frac{2}{1000} \times 2 \times 9.8 \times 0.2$

$= 3.92 \times 10^{-3}$ J = 3.92 mJ

Q.18 (2)

In the stable equilibrium, a body has minimum potential energy.

Q.19 (1)

Here $V(x) = (x^2 - 3x)$J

For a conservative field, force, $F = -\frac{dV}{dx}$

$$\therefore F = -\frac{d}{dx}\left(x^2 - 3x\right) = -(2x - 3) = -2x + 3$$

At equilibrium position, F = 0

$$\therefore -2x + 3x = 0 \text{ or } x = \frac{3}{2}\text{ m} = 1.5\text{m}$$

Q.20 (4)

Condition for vertical looping $h = \frac{5}{2}r = 5\text{cm}$

$\therefore$ r = 2 cm

JEE-MAIN
OBJECTIVE QUESTIONS

Q.1 (2)
Work done by centripetal force is always zero, because force and instantaneous displacement are always perpendicular.

$W = \vec{F}.\vec{s} = Fs\cos\theta = Fs\cos(90^o) = 0$

Q.2 (3)
$25 = 5 \times 10 \times \cos\theta$ so $\theta = 60^o$

Q.3 (2)
Work done does not depend on time.

Q.4 (2)
$W = (2000 \sin 15^o) \times 10 = 5176.8$ J

Q.5 (3)
$W = 20 \times 10 \times 20 \times 0.25 = 1000$ J

Q.6 (1)
$W = \vec{F}\,.\,(\vec{r}_2 - \vec{r}_1) = 100$ J

Q.7 (3)

$S_1 = \frac{1}{2} g 1^2,\ s_2 = \frac{1}{2} g 2^2,\ S_3 = \frac{1}{2} g 3^2$

$S_2 - S_1 = \frac{1}{2} g 3,\ S_3 - S_2 = \frac{1}{2} g 5$

$W_1 = (mg) S_1,\ W_2 = (mg)(S_2 - S_1),\ W_3 = (mg)(S_3 - S_2)$

$W_1 : W_2 : W_3 = 1 : 3 : 5$

Q.8 (1)

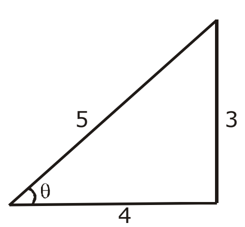
$T = mg + ma,\ S = \frac{1}{2} at^2$

$W_T = T \times S$

$= \frac{m(g+a)at^2}{2}$

Q.9 (4)
Displacement of surface point (where force acts) = 0 hence W = 0

Q.10 (2)



$w = mgh,\ \cos\theta = 4/5$
$= 10 \times 9.8 \times 3 = 294$ joule

Q.11 (2)
$W_a + W_c = \Delta K = 0,$

$W_a - mg\left(\frac{\ell}{2} - \frac{\ell}{2}\cos 60^o\right) = 0$

$W_a = \frac{mg\ell}{4} = (0.5)(10)\left(\frac{1}{4}\right) = \frac{5}{4}$ J.

Q.12 (3)

$F = K_1x_1,\ x_1 = \frac{F}{K_1},\ W_1 = \frac{1}{2} K_1 x_1^2 = \frac{F^2}{2K_1}$

similarly $W_2 = \frac{F^2}{2K_2}$ since $K_1 > K_2$, $W_1 < W_2$

Q.13 (4)
W_1 = work done by spring on first mass
W_2 = work done by spring on second mass
$W_1 = W_2 = W$ (say)
$W_1 + W_2 = U_i - U_f$

$2W = 0 - \frac{1}{2} Kx^2$

$W = -\frac{Kx^2}{4}$

Q.14 (4)

$W_F = \int\left(\frac{K}{S}\right)ds = K \ln s + C$ Ans : (D)

Q.15 (1)

$W = \int_0^1 F\,dx = \frac{1}{6}$ J

Q.16 (3)
Let $\vec{r} = dx\hat{i} + dy\hat{j},\ F = 3x\hat{i} + 4\hat{j}$

$w = \int(3x\hat{i}+4\hat{j}).(dx\hat{i}+dy\hat{j})$

$= \int_{2m}^{3m} 3x\,dx + \int_{2m}^{0} 4dy = \left[\frac{3x^2}{2}\right]_{2m}^{3m} + [4y]_{3m}^{0}$

$= \left[\frac{3\times 9}{2} - \frac{3\times 2^2}{2}\right] + [0-12] = -4.5$ J

Q.17 (3)

A = area under the curve $= m\int_0^v v\frac{dv}{dx}dx = \frac{mv^2}{2}$

$\frac{100\times 11}{2} = \frac{mv^2}{2} = mgy_{max}$

$\therefore y_{max} = 11$ m

Q.18 (2)

$2\ K.E_{man} = K.E._{boy}$

$2 \times \frac{1}{2} M \times v_{man}^2 = \frac{1}{2}.\frac{M}{2} v_{boy}^2$

$V_{man} = \frac{v_{boy}}{2}$ ...(i)

$\Rightarrow \frac{1}{2} M (v_{man}+1)^2 = \frac{1}{2}.\frac{M}{2} v_{boy}^2$

$\Rightarrow (v_{man}+1)^2 = \frac{V_{boy}^2}{2} \Rightarrow v_{man} = (\sqrt{2}+1)\, m/sec$

Q.19 (1)

$KE = \frac{P^2}{2m} = 1$

Q.20 (2)

$a = \frac{F}{m}, S = \frac{1}{2}\left(\frac{F}{m}\right)t^2, W_F = FS = F\left(\frac{Ft^2}{2m}\right)$

Q.21 (4)

Area under curve $= \frac{1}{2}$ (4) (20) = 40 J

W = work done by resistive force F = – 40 J

$-40 = K_f - K_i$, $K_i = 50$ J, so $K_f = 50 - 40 = 10$ J

Q.22 (4)

$W = \text{area} = 80 = \frac{1}{2}(0.1)\, u^2 - 0$,

so u = 40 m/s

Q.23 (1)

$h = \frac{1}{2} gt^2$, $W = mgh = mg\ \frac{gt^2}{2}$, $W = K_f - K_i$

$\frac{mg^2 t^2}{2} = K_f - \frac{1}{2} mu^2$, $K_f = \frac{1}{2} mu^2 + \frac{mg^2 t^2}{2}$

Hence Ans. is (A)

Q.24 (4)

$V = O + aT$, $a = \frac{V}{T}$, velocity $= O + at = \frac{Vt}{T}$

$K.E = \frac{1}{2}(m)\left(\frac{Vt}{T}\right)^2$

Q.25 (1)

$E = \frac{1}{2} mV^2$, $\frac{dE}{dV} = mV = p$

Q.26 (4)

$W_G = \frac{1}{2} mV_f^2 - \frac{1}{2} mV_i^2, mg\,h = \frac{1}{2} mV_f^2 - \frac{1}{2} mV^2$,

So V_f is free from direction of V.

Q.27 (1)

$V_0 = at_0 \qquad \Rightarrow a = \frac{v_0}{t_0}$

$\therefore v = \frac{v_0}{t_0}.t \qquad \Rightarrow w = \Delta k = k_f - k_i$

$\Rightarrow \frac{1}{2} M \frac{v_0^2}{t_0^2}.t^2$

Q.28 (2)

$-F x = 0 - \frac{1}{2} m (2)^2$

and $-FS = 0 - 2\left[\frac{1}{2} m (2)^2\right]$

So $\frac{S}{x} = 2$, $S = 2x$

Q.29 (4)

$F\ 80 = \frac{1}{2} mV^2$, $FS = \frac{1}{2} m (2V)^2$

So $\frac{s}{80} = 4$, $S = 4\ (80)$

Q.30 (A)

$V\frac{dV}{dx} = -Kx, \left[\frac{V^2}{2}\right]_u^v = -\left[\frac{Kx^2}{2}\right]_0^x$

$V^2 - u^2 = -\ Kx^2$

$\frac{1}{2} mu^2 - \frac{1}{2} mV^2 = \frac{1}{2} mK\, x^2$

Loss α x^2

Q.31 (1)

$W_G + W_f = 0 - 0$

$10 \times 1 + W_f = 0$

$10 - \mu mg\, x = 0$

$10 = (.2)(10)\, x$, $x = 5$ m

Q.32 (2)

Maximum velocity will be at Mean Position

Where $F_{net} = 0 \Rightarrow mg = Kx$

$1 \times 10 = 2 \times 100 \times x \quad \Rightarrow \quad x = 5$ cm

$\therefore h = 20 - 5 = 15$ cm

Q.33 (1)

$w = \frac{1}{2}k(x_2^2 - x_1^2)$

$= \frac{1}{2}\,10\,(6^2 - 4^2) = 100$ N cm

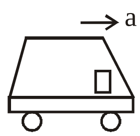
= 1 joule

Q.34 (1)

(1) (mg)1 – mg/2 = mv² /2, $v = \sqrt{g}$;

$d = v\sqrt{2h/g} = \sqrt{g}\sqrt{\frac{2(0.5)}{g}} = 1$ m

Q.35 (3)



P = F.V = (R + ma) V

Q.36 (4)

Average power $= \frac{100 \times 9.8 \times 50}{50} = 980$ J/s

Q.37 (4)

V = 0 + at, F – μ mg = ma , F = μmg + ma,

P = (μmg + ma) at

Q.38 (3)

$P = \bar{F}.\bar{v} = 50 - 30 + 120 = 140$ J

Q.39 (2)

P = TV = 4500 × 2 = 9000 W = 9KW

Q.40 (2)

$P_1 = 80$ gh/15 , $P_2 = 80$ gh/20

$\frac{P_1}{P_2} = \frac{20}{15} = \frac{4}{3}$

Q.41 (3)

Given m = 12000 kg, v = 4 m/sec & t = 40 sec

$P_{avg} = \frac{\frac{1}{2}mv^2}{t} = \frac{\frac{1}{2} \times 12000 \times 4^2}{40} = 2400W = 2.4$ kW

Q.42 (2)

Q.43 (3)

Follows from definition

Q.44 (3)

Potential energy depends upon positions of particles

Q.45 (1)

$U_i + 0 = U_f + \frac{1}{2}mv^2$

$U_i - U_f = \frac{1}{2}mv^2$

$U = \frac{1}{2}mv^2$

$m = \frac{2U}{v^2}$

Q.46 (3)

$\frac{1}{2}mu^2 = mgh,\ u^2 = 2gh$ ....(i)

$mg\left(\frac{3h}{4}\right) + K.E. = mgh$

$K.E. = \frac{mgh}{4}$

$\frac{K.E.}{P.E.} = \frac{mgh/4}{3mgh/4} = \frac{1}{3}$

Q.47 (2)

$W_F + W_S = 0,\ \ W_F - \Delta U = 0\ ,\ \ W_F = \Delta U = E$

$E = \frac{1}{2}K_A x_A^2,\ Fx_A = \frac{1}{2}K_A x_A^2$

$\frac{2F}{K_A} = x_A\ ,\ \frac{2F}{K_A} = \sqrt{\frac{2E}{K_A}}\ ,\ K_A = \frac{2F^2}{E}$ ...(i)

similarly $K_B = \frac{2F^2}{E_B}$, ∵ $K_A = 2K_B$ ∴

$\frac{2F^2}{E} = 2\left(\frac{2F^2}{E_B}\right)$

∴ $E_B = 2E$

Alter :

$F = K_A x_A = K_B x_B$

$E_A = \frac{1}{2}K_A x_A^2$

$E_B = \frac{1}{2}K_B x_B^2$

$\frac{E_A}{E_B} = \left(\frac{K_A}{K_B}\right)\left(\frac{x_A}{x_B}\right)^2$

$\frac{E_A}{E_B} = 2\left(\frac{1}{2}\right)^2 = \frac{1}{2}$

Q.48 (4)

$$\frac{1}{2}K(0.3)^2 = 10 \Rightarrow K = \frac{20}{0.09} = \frac{2000}{9}$$

work done = $\frac{1}{2}.\frac{2000}{9}[(0.45)^2 - (0.3)^2]$ = 12.5 J

Q.49 (1)

$u = x^2 - 3x$, x = 0, x = 2

$(u_i)_{x=0} = 0$, $(u_f)_{x=2} = 4 - 6 = -2$

$\Delta k = -\Delta u = 2$ joule

Q.50 (3)

$100 = \frac{1}{2}K(2cm)^2$, $E = \frac{1}{2}K(4cm)^2$

so $\frac{E}{100} = 4$, E = 400 J

$\therefore$ E – 100 = 300 J

Q.51 (1)

$$\frac{1}{2}K_2x^2 + \frac{1}{2}K_1x^2 = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{K_1 + K_2}{m}}\ x$$

Q.52 (4)

$4\ J = \frac{1}{2}k(2)^2$ ...(1)

$X\ J = \frac{1}{2}k(10)^2$

....(2)

from equation (1) & (2)

x = 100 J

Q.53 (3)

$$\mu mg = Kx,\ U = \frac{1}{2}Kx^2 = \frac{(\mu mg)^2}{2K}$$

Q.54 (3)

For m, N cos θ = mg

For M , N sin θ = kx

so $\tan\theta = \frac{Kx}{mg}$

so $\frac{1}{2}Kx^2 = \frac{(mg\tan\theta)^2}{2K}$

Q.55 (1)

$$T = Kx,\ U = \frac{1}{2}Kx^2 = \frac{1}{2}K\left(\frac{T}{K}\right)^2 = \frac{T^2}{2K}$$

Q.56 (1)

$$mg\left(h + \frac{3mg}{K}\right) = \frac{1}{2}K\left(\frac{3mg}{K}\right)^2$$

Q.57 (4)

$(W.D)_{\text{by friction}} + (W.D)_{\text{by spring}} = \Delta k = k_f - k_i = 0 - k_i$

$$-0.25 \times 1 \times 10 \times 4 - \frac{1}{2} \times 2.75 \times 4^2 = -\frac{1}{2}\times 1 \times v^2$$

v = 8 m/s

Q.58 (3)

$$\frac{du}{dr} = 0,\ -\frac{2a}{r^3} + \frac{b}{r^2} = 0,\ r = \frac{2a}{b}$$

Q.59 (2)

$$\left.\frac{dU}{dx}\right|_{x=A} = -ve,\quad \left.\frac{dU}{dx}\right|_{x=B} = +ve$$

So, F_A = positive , F_B = negative

Q.60 (3)

$F = -\frac{dU}{dx} = 0$ at B and C

Q.61 (1)

Only in (A), U is minimum for some value of r

Q.62 (1)

$W_C = W_C + W_C = 5 + 2 = 7$

$P \to R$ $P \to Q$ $Q \to R$

Q.63 (1)

$\frac{\partial U}{\partial x} = \cos(x + y)$,

$\frac{\partial U}{\partial y} = \cos(x + y)$

$\bar{F} = -\cos(x + y)\,\hat{i} - \cos(x + y)\,\hat{j}$

$= -\cos(0 + \frac{\pi}{4})\,\hat{i} - \cos(0 + \frac{\pi}{4})\,\hat{j}$

$\Rightarrow |\bar{F}| = 1$

Q.64 (1), (2), (3)

From work energy theorem

$W_C + W_{nC} = \Delta K$, $W_C = -\Delta U$, $W_{nC} - \Delta U = \Delta K$

Q.65 (1)

Area under force vs displacement gives work and area above x-axis taken as positive while area below x-axis taken as negative.

W_{net} = 10 × 1 +20× 1- 20 ×1+10×1= 20 erg.

Q.66 (1)

$2x^2 - 3x - 2 = 0$

$$x = \frac{3 \pm \sqrt{9+16}}{4} = \frac{3 \pm 5}{4} \Rightarrow x = -\frac{1}{2}, 2$$

$$\frac{dF}{dx} = -\frac{d^2u}{dx^2} = 4x - 3 \Rightarrow \frac{d^2u}{dx^2} = 3 - 4x$$

$$\Rightarrow \left(\frac{d^2u}{dx^2}\right)_{x=-\frac{1}{2}} = 3 + 4 \times \frac{1}{2}$$

= (5) > 0 (stable)

Q.67 (1)

(1) $mg\frac{\ell}{2} = \frac{1}{2}mv^2$

$v = \sqrt{g\ell}$

Q.68 (3)

Initially be in contact with the inner wall and later with the outer wall.

Q.69 (2)

For light rod

$v_{top} = 0$

Using energy conservation

$\frac{1}{2}mv^2 + 0 = 0 + mg\ell$

$v = \sqrt{2g\ell}$

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OBJECTIVE QUESTIONS

Q.1 **(C)**

f = frictional force = mg sin θ

displacement of point of application in t second = vt (↓)

$W_f = [(mg\sin\theta)\sin(180-\theta)](vt) = -mgvt\sin^2\theta$

Q.2 **(C)**

$W_{agent} + W_G = \Delta K = 0$

$W_{agent} = -W_G$, But W_G is independent of the path joining initial and final position. W_G is independent of time taken.

Q.3 **(C)**

$$W.D. = \int \vec{F}.\vec{ds}$$

$$= K\int[(y\hat{i} + x\hat{j}).(dx\hat{i} + dy\hat{j})]$$

$$= K\int(ydx + xdy)$$

$$= K\int_{(1,5)}^{(3,5)} d(xy) = 20K$$

Q.4 **(B)**

$F = T$, $W_F + W_G = 20$

$W_T = 20 \Rightarrow 20 + W_G = 20 \Rightarrow W_G = 0$

which is not possible.

Q.5 **(A)**

$W_f + W_G = \Delta K$

$-\mu mgd - mgh = 0 - \frac{1}{2}m v_0^2$

$\mu gd + gh = \frac{1}{2}(v_0^2)$

(0.6) (10) d + 10(1.1) = 18 d = $\frac{7}{6}$ = 1.1666 ≈ 1.17

Q.6 **(C)**

$W_G - W_f = 0$, $mgh = \mu mg\ell$

$h = \mu\ell$

$h = (0.2)\ell \Rightarrow \ell = \frac{1.5}{0.2} = 7.5$

ℓ = 7.5 m = (3 + 3 + 1.5)m

Q.7 **(A)**

$W_S + W_f = \Delta K$

$-\Delta U + W_f = -K_i$

$-U_f - \mu mgx = -K_i$

$\frac{1}{2}Kx^2 + \mu mgx = \frac{1}{2}mu^2$

$100x^2 + 2(0.1)(50)(10)x = 50 \times 4$

$x^2 + x - 2 = 0$

x = 1 m

Q.8 **(A)**

$v = \beta\sqrt{s}$

$\frac{ds}{dt} = \beta\sqrt{s}$,

$$\int_0^s \frac{ds}{\sqrt{s}} = \beta\int_0^t dt$$

$2\sqrt{s} = \beta t$

$\sqrt{s} = \beta t/2$ ....(1)

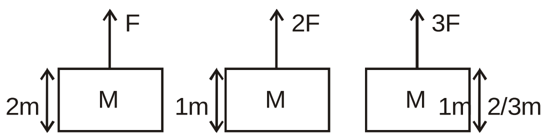
W = workdone by all the forces = ΔK

$$= \frac{1}{2}mv^2 = \frac{1}{2}m\beta^2 s = \frac{1}{2}m\beta^2\left(\frac{\beta^2 t^2}{4}\right)$$

Q.9 **(A)**

$$\frac{1}{2}(100)\left(\frac{10}{100}\right)^2 = \left(\frac{250}{1000}\right)(10)\left(\frac{H}{100}\right),\ H = 20\text{ cm.}$$

Q.10 **(C)**



Apply work energy theorem

$W_F + W_{mg} = \Delta K = K_f - K_i\ (K_i = 0)$

Case I : $F(2) - mg \times 2 = K.E.$

Case II : $2F(1) - mg \times 1 = K.E.$

Case III : $3F\left(\frac{2}{3}\right) - mg \times \left(\frac{2}{3}\right) = K.E.$

In case III K.E. is maximum.

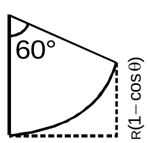
Q.11 **(C)**

$W_R + W_G = 0,\ -Rd + mg(h + d) = 0$

$R = mg\left(1 + \frac{h}{d}\right)$

Q.12 **(B)**

W = Rθx F cos 0° (by the force)



$= 10 \times \frac{\pi}{3} \times 200$

Work done by g = MgR (1 – cos 60°)

$= \frac{gRM}{2}$

$K.E. = RF\theta - \frac{gRM}{2}$

$$\frac{1}{2}MV^2 = 10 \times \frac{\pi}{3} \times 200 - \frac{10 \times 10 \times 10}{2}$$

$$v^2 = 2 \times \frac{\pi}{3} \times 200 - 50$$

V = 17.32 m/s

Q.13 **(C)**

v = at

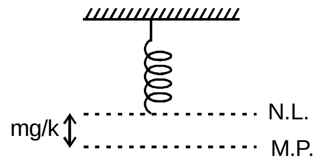
$= 10\sqrt{3}$ m/s

In ground frame

W.D. by gravity + W.D. by normal = Δk

$$0 + W.D._N = \frac{1}{2} \times 1 \times (10\sqrt{3})^2 = 150J$$

Q.14 **(B)**



$K = \frac{mg}{a}$(Given)

$$\frac{1}{2} \times m \times v^2 + \frac{1}{2}k\left(\frac{mg}{k}\right)^2 = mg\left(\frac{mg}{k}\right)$$

$$\frac{1}{2} \times m \times v^2 + \frac{1}{2} \times \frac{mg}{a} \times \frac{m^2g^2}{m^2g^2} \times a^2 = \frac{m^2g^2}{mg} \times a$$

$$\frac{1}{2}mv^2 + \frac{1}{2}mga = mga$$

$v^2 = ga$

$$K.E. = \frac{1}{2}mv^2 = \frac{mga}{2}$$

Q.15 **(C)**

$$P = FV = m\left(\frac{dv}{dt}\right)v$$

$$P\int_0^t dt = m\left[\frac{v^2}{2}\right]_0^v$$

$$Pt = \frac{mv^2}{2},\ v^2 = \frac{2Pt}{m},\ v = \frac{ds}{dt} = \sqrt{\frac{2P}{m}}\sqrt{t}$$

$$\int_0^t ds = \sqrt{\frac{2P}{m}}\int_0^t \sqrt{t}\,dt\ ; s \propto t^{3/2}$$

Q.16 **(B)**

On comparing

$F \propto V$

$F = kV$

$P = F.V = kV^2$

$\Rightarrow$ Now $2P = KV'^2$

$2 \times kv^2 = kV'^2$

$\Rightarrow$ $V'^2 = 2V^2$

$V' = \sqrt{2}V$

Q.17 **(B)**

$$\frac{dW}{dt} = \frac{d.K.E.}{dt}(K.E = 2t^2)$$

$$\Rightarrow \quad P = \left(\frac{dK.E.}{dt}\right)_{at\, t=2s} = 4t = 8watt$$

Q.18 **(B)**

$W_{ext} + W_C + W_{ps} = \Delta K$

Q.19 **(A)**

Total energy = E = K.E + P.E.
When speed of the particle is zero.
i.e., K.E = 0
$\Rightarrow$ U(x) = E

Q.20 **(A)**

Angle of Inclination

Q.21 **(D)**

Only Conservative force (mg) is act.
So E.C. is done only two points
(1 and 2)

Q.22 **(B)**

K.E. + P.E. = constant = C (say)

$K - mg\,(tu \sin\theta - \frac{1}{2}gt^2) = C$

$K = mg\,[tu \sin\theta - \frac{1}{2}gt^2] + C$ [= parabolic]

$C \neq 0$ so answer is (B)

Q.23 **(C)**

$\frac{dU}{dx}$ = positive constant

For x < a, F = negative constant and for x > a, F = 0
so, ans. (C)

Q.24 **(A)**

K.E. + P.E. = positive constant C
E + U = C, E + mgh = C, E = – mgh + C
and U = mgh,
So, answer (A)

Q.25 **(C)**

$$E = \frac{p^2}{2m}, \quad (\sqrt{E})\left(\frac{1}{P}\right) = \frac{1}{\sqrt{2m}} = \text{constant}$$

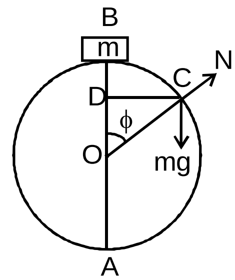
Rectangular hyperbola (C)

Q.26 **(B)**

At $x = x_2$, as x increases, F acts along negative x-direction.
So, answer (C)

Q.27 **(B)**

$$mg\cos\phi - N = \frac{mv^2}{R}$$



$N = m(g\cos\phi - \frac{v^2}{R})$ ...(i)

$\because N = 0$

$\Rightarrow \cos\phi = \frac{v^2}{Rg}$ ...(ii)

By energy conservation

$$\frac{1}{2}mv^2 = mg(R - R\cos\phi) \Rightarrow v^2 = 2Rg(1-\cos\varphi)$$

Using (i) & (ii) $\cos\phi = \frac{2}{3}$

height from highest Point = BD = $R(1 - \cos\phi)$

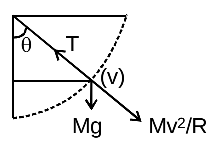
$h = R\left(1 - \frac{2}{3}\right) = \frac{R}{3}$ Ans.

Q.28 **(C)**

$$\sqrt{5Rg} = \sqrt{5 \times 2.5 \times 10} = 5\sqrt{5} > 10\,m/s$$

$\therefore N_2$ will be zero in part A, D, C at some point

Q.29 **(A)**

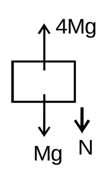


$$T = \frac{Mv^2}{R} + Mg\cos\theta \quad \Rightarrow \quad MgR\cos\theta = \frac{1}{2}Mv^2$$

$$\Rightarrow \quad Mgh = \frac{1}{2}Mv^2$$

$$\Rightarrow \quad T = \frac{2Mgh + Mgh}{R} \quad \text{(Straight line)}$$

Q.30 **(C)**



$$2MgR = \frac{1}{2}Mv^2 \Rightarrow 2\sqrt{gR} = V$$

$$\frac{mv^2}{R} = mg + N \quad \Rightarrow N = 3\ mg$$

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MCQ/COMPREHENSION/COLUMN MATCHING

Q.1 **(A,C,D)**

$dW_F = \vec{F}.d\vec{s}$, if $\vec{F}$ perpendicular to $d\vec{s}$ then

$dW_F = 0$, $d\vec{s}$ is displacement of point of application

of force, $\vec{v} = \frac{d\vec{s}}{dt}$.

(A), (C), (D) are true.

Q.2 **(A, B, C)**
Follows from work energy theorem.

Q.3 **(A, B, C)**
This can be explained by two blocks problem.

Q.4 **(A,B)**
(A) The spring initially compressed and finally in its N.L.
(B) Initially stretched and then in its N.L.

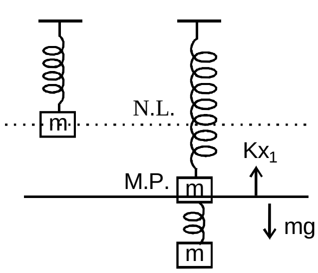
Q.5 **(B, D)**

$dW_F = \vec{F}.d\vec{s} = dk > 0 \Rightarrow |\vec{F}||d\vec{s}|\cos\theta > 0$
$\Rightarrow 0 < \theta < 90^o$

$p = \sqrt{2m(K.E.)}$, K.E. $\uparrow$ so p $\uparrow$.

Q.6 **(A, B, C)**
W = ΔK, 0 = ΔK, k remains constant, speed remains constant.
W = ΔK, 0 = ΔK, k

Q.7 **(B,C,D)**



M.P. $x_1 = \frac{mg}{k}$

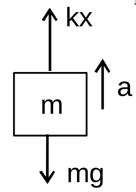
But block further move downward due to inertia. So descending through distance

$$x = \frac{2mg}{k}$$

at M.P. at $\frac{x}{2} \Rightarrow F_{net} = 0$;

so a = 0

n



at lower most point

$$k\left(\frac{2mg}{k}\right) - mg = ma \Rightarrow a = g$$

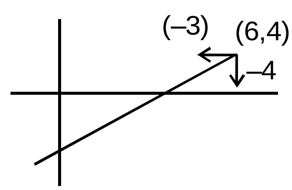
Q.8 **(B, C)**
W = ΔK > 0 ⇒ K (= kinetic energy) increases
$p = \sqrt{2mk}$, p ↑ as k↑.

Q.9 **(B, C)**
B and C holds when a ball moves in upward direction.

Q.10 **(A,B,C)**
Given U = 3x + 4y
Initially particle at rest at (6,4)
So K.E = 0
E_{total} = P.E = 3 × 6 + 4 × 4 = 34 J

$$F = -\frac{\partial U}{\partial y}\hat{i} - \frac{\partial U}{\partial y}\hat{j} = -3\hat{i} - 4\hat{j}$$

$$a = -3\hat{i} - 4\hat{j} \Rightarrow |a| = 5\,m/s^2$$



Let us assume particle crosses y axis after time t

$$x - 6 = -\frac{1}{2}\times 3\times t^2$$

at y axis ⇒ x = 0 ⇒ t = 2 sec

So $y - 4 = -\frac{1}{2}\times 4\times (2)^2 = -8$

y = – 4m

(P.E.) at y = – 4 and x = 0

is $U_{(y=-4,\ x=0)} = -16$ J

So. K.E. = T.E. – U

$$\frac{1}{2}MV^2 = 34 - (-16) = 50$$

$V^2 = 100 \Rightarrow V = 10$ m/s

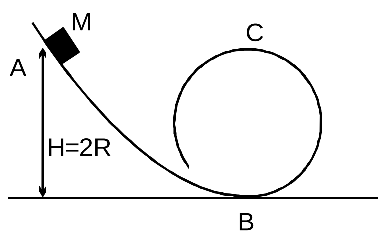
Q.11 **(B)**

Q.12 **(B)**
When the particle is released at $x = 2 + \Delta$ it will reach the point of least possible potential energy (–15 J) where it will have maximum kinetic energy.

$$\therefore \quad \frac{1}{2}m v_{max}^2 = 25 \Rightarrow \quad v_{max} = 5 \text{ m/s}$$

Q.13 **(D)**

Q.14 **(B,D)**



E.C between point A and B

$$Mg\,(2R) = \frac{1}{2}MV^2$$

$$V = \sqrt{4gR} < \sqrt{5gR}$$

$$V = \sqrt{4gR} > \sqrt{2gR}$$

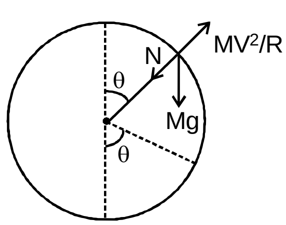
So, doesn't complete vertical circle and break off at a height (R < H < 2R)

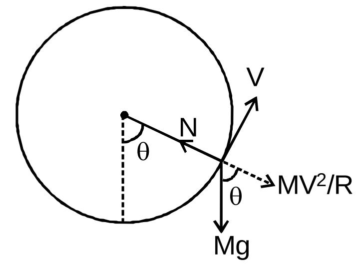
Q.15 **(A,B,D)**

$$N = \frac{Mv^2}{R} + Mg\cos\theta$$

N_{max} at $\theta = 0°$
N is zero only
$\theta \geq \pi/2$ because in this

$$N = \frac{MV^2}{R} - Mg\cos\theta$$





Q.16 **(C)**
To complete vertical circle

speed at point B ≥ $\sqrt{5gR}$

So. E.C.

$$MgH = \frac{1}{2}M(5gR)$$

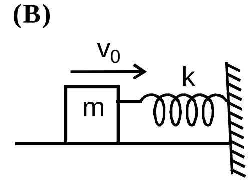
$$H = \frac{5R}{2} = 2.5\text{ R}$$

Q.17 (A)
Q.18 (D)
Q.19 (A)

Q.20 **(C)**
Friction is present
∴ Mechanical energy is not conserved
But work energy principle conserved
Due to extrenal friction force is working on the block.

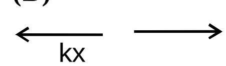
Q.21 **(C)**
The block will come to rest when work done by friction becomes equal to the change in energy stored in spring.

Q.22 **(B)**



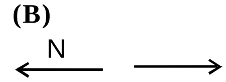
$$-\frac{1}{2}mv_0^2$$

Q.23 **(B)**



$$-\frac{1}{2}mv_0^2$$

Q.24 **(B)**



Net work done = $-\frac{1}{2}mv_0^2$

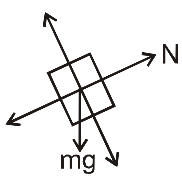
Q.25 **(B)**

Q.26 **(C)**
Velocity of block with respect to observer B is zero so K.E of block = 0

Q.27 **(B)**
P.E ↑
Due to +ve work done by N

Q.28 (A) p, r (B) q, s (C) q, r (D) p
The displacement of A shall be less than displacement L of block B.
Hence work done by friction on block A is positive and its magnitude is less than μmgL.
And the work done by friction on block B is negative and its magnitude is equal to μmgL.
Therefore workdone by friction on block A plus on block B is negative its magnitude is less than μmgL.
Work done by F is positive. Since F> μmg, magnitude of work done by F shall be more than μmgL.

Q.29 (A) q, s (B) p, s (C) r, s (D) p, s
(A) The FBD of block is



Angle between velocity of block and normal reaction on block is obtuse
$\therefore$ work by normal reaction on block is negative.
As the block fall by vertical distance h,

from work energy Theorem
Work done by mg + work done by N = KE of block

$\therefore$ |work done by N| = mgh – $\frac{1}{2}$mv^2

$\because$ $\frac{1}{2}$mv^2 < mgh

$\therefore$ |work done by N| < mgh
(B) Work done by normal reaction on wedge is positive
Since loss in PE of block = K.E. of wedge + K.E. of block
Work done by normal reaction on wedge = KE of wedge.
$\therefore$ Work done by N < mgh.
(C) Net work done by normal reaction on block and wedge is zero.
(D) Net work done by all forces on block is positive, because its kinetic energy has increased.
Also KE of block < mgh
$\therefore$ Net work done on block = final KE of block < mgh.

NUMERICAL VALUE BASED

Q.1 [21 J]

Q.2 [54 sec]

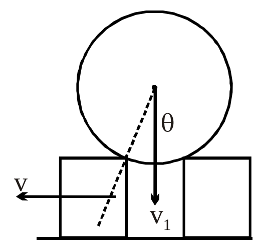
Q.3 [8]
Applying work energy theorem when block comes down by x = 10 cm

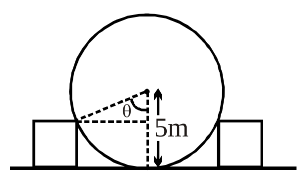
$$w_{mg} + w_{sf} + w_f = 0$$

$$mgx \sin\theta - \frac{1}{2}kx^2 - \mu\, mg\, x \cos\theta = 0$$

on solving it gives $\mu = \frac{1}{8}$ Ans.

Q.4 [5 m/s]
Along normal their velocity are same.





$v_1 \cos\theta = v \sin\theta$
at instant of touching ground.

$$\cos\theta = \frac{2.5}{5} = \frac{1}{2} \Rightarrow \theta = 60° \Rightarrow \frac{v_1}{2} = \frac{v\sqrt{3}}{2}$$

$$w_g = \Delta k \Rightarrow mg \times 2.5 = \frac{1}{2}mv_1^2 \Rightarrow 25 = \frac{v_1^2}{2} + \frac{3}{2} \times \frac{v_1^2}{3} \Rightarrow v_1 = 5 \text{ m/s}$$

Q.5 [9600]
When the spring is compressed by 1.00 m, the sledge moves further down vertically by
1.00 × sin 30° = 0.50 m.
Conservation of energy (gravitational potential energy and elastic potential energy) :

$$120 \times 10 \times (3.50 + 0.50) = \frac{1}{2} k \times 1.00^2$$

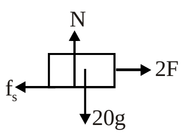
$k = 9600\ Nm^{-1}$

Q.6 [60]

$$0.8 \times 30 \times 10^3 \times 30 = \frac{1}{2}(400)v^2 = 60 \text{ m/s}$$

Q.7 [1920]

$t_{max} = \mu N = 0.4 \times 20g = 80$



F = 40 t

Sliding starts when

$F = f_{max}$

80t = 80 $\therefore$ t = ⊥s

$$\int 2F\,dt - \int f_k dt = mv\ \ 0$$

$$\Rightarrow \int_1^3 80t\ dt - 80\int_1^3 dt = 20\ V$$

$\therefore$ v = 8m/s

$P = \vec{F} \cdot \vec{v} = 40t \times 2v = 40 \times 3 \times 16 = 1920$

Q.8 [500]

$$a = \frac{4-2}{4+2} g = \frac{g}{3}$$

$$v_1 = at_1 = \frac{2g}{3}$$

$v_2 = at_2 = g$

$$\Delta k_a = \frac{1}{2} M\ (v_2^2 - v_1^2) = \frac{1}{2} \times 6 \left[g^2 - \frac{4g^2}{9}\right] =$$

$$\frac{5g^2}{3} = \frac{500}{3} \text{ J}$$

Q.9 [9000]

$$-FS = \frac{1}{2}mv^2 - \frac{1}{2}mv^2$$

$$v = \sqrt{v^2 - \frac{2Fs}{m}} = 9000 \text{ m/s}$$

Q.10 [640 kJ]

$$WD_A = \frac{1}{2} m_1 v_1^2 = 960 \text{ kJ}$$

$$WD_B = \frac{1}{2} m_2 v_2^2 = 1600 \text{ kJ}$$

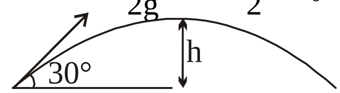
____

$WD_B - WD_A = 640$ kJ

Q.11 [25]

U = mgh

$$= mg\frac{v_0^2 \sin^2\theta}{2g} = \frac{1}{2} m v_0^2 \sin^2\theta$$



$$= 100 \times \frac{1}{4} = 25 \text{ J}$$

Q.12 [20]

$$\sqrt{a^2 + b^2} = 0.25$$

$$\delta = \sqrt{a^2 + b^2} - l = 0.05 \text{ m}$$

$$2 \times \frac{1}{2} k\delta^2 = \frac{1}{2} mv^2$$

$v^2 = 2gh$

$kd^2 = mgh$

$400 \times (0.05)^2 = 5 \times 10^{-3} \times 10 \times h \Rightarrow h = 20$ m

Q.13 [450]

$$a = \frac{(m_1 - m_2)}{m_1 + m_2} g = \frac{4-1}{4+1} g = 6 \text{ m/s}^2$$

$$\Delta k = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 - \frac{1}{2} m_1 u_1^2 - \frac{1}{2} m_2 u_2^2$$

$= m_1 gh_1 + m_2 gh_2$

$= (m_1 - m_2)gh$

$$h = 0 + \frac{1}{2} a\,(2n - 1)$$

$$= \frac{1}{2} \times 6 \times (2 \times 3 - 1) = 15 \text{ m}$$

$= 3 \times 10 \times 15 = 450$ J

Q.14 [75 sec.]

$$p = \frac{mgh}{t} = \frac{300 \times 10 \times 24}{t} = 960$$

$$t = \frac{300 \times 10 \times 24}{960} = 75 \text{ sec.}$$

Q.15 [6]

$$F = \frac{\partial U}{\partial x}\hat{i} - \frac{\partial U}{\partial y}\hat{j}$$

$$\frac{-\partial U}{\partial x} = -6x^2 + 8xy = 6 \times 3^2 + 8 \times 3 \times 2$$

$$\frac{-\partial U}{\partial y} = +4x^2 - 12y = +4 \times 3^2 - 12 \times 2 = +12$$

A + B = 6

Q.16 [10]

When the maximum speed is achieved, the propulsive force is equal to the resistant force. Let F be this propulsive force, then

F = aV and FV = 600 W

Eliminating F, we obtain

$$V^2 = \frac{400}{a} = 100 \text{ m}^2/\text{s}^2$$

and the maximum speed on level ground with no wind

v = = 10 m/s

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Q.1 (B)

mgh = 300 × 10 × 6

$$P_i = \frac{mgh}{t} = \frac{300 \times 10 \times 6}{60} = 300\,W$$

P_0 = 750 W

$$\eta = \frac{300}{750} \times 100 = 40\%$$

Q.2 (D)

According to work-energy principle

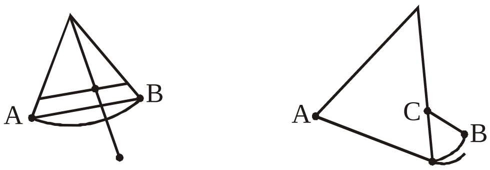
$W_C + W_{nc} + W_{ext} = \Delta KE$

$$\int_4^8 F dx = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$\frac{1}{2} \times 3 \times 8 - \frac{1}{2} \times 1.5 \times 4 = \frac{1}{2} \times \frac{1}{2}\left[v_f^2 - (3.16)^2\right]$$

v_f = 6.8 m/s

Q.3 (A)



According to law of conservation of mechanical energy

$K_i + U_i = K_f + U_f$

$0 + U_i = 0 + U_f$

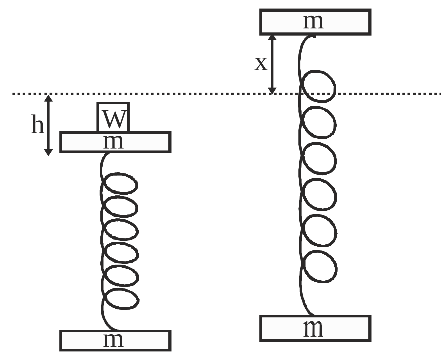
$h_i = h_f$

Point D is at line AB

Q.4 (B)

For lower block +ve lift, kx ≥ mg

$$\Rightarrow x \geq \frac{mg}{k}$$



W/E theorm

$$-mg(h + x) + \left(\frac{1}{2}kh^2 - \frac{1}{2}kx^2\right) = 0 - 0$$

$$\Rightarrow -mgh - \frac{m^2g^2}{k} + \frac{1}{2}kh^2 - \frac{1}{2}\frac{m^2g^2}{k} = 0$$

$$\frac{kh^2}{2} - mgh - \frac{3m^2g^2}{2k} = 0$$

$$h = \frac{mg \pm \sqrt{m^2g^2 + 3m^2g^2}}{k}$$

$$= \frac{mg \pm 2mg}{k} = \frac{3mg}{k}, \frac{-mg}{k}$$

$$\therefore h = \frac{3mg}{k}$$

W + mg = kh

W + mg = 3mg

W = 2mg

Q.5 (C)

$W_f + W_{mg} = \Delta K.E.$, ($\Delta$ K.E. = 0)

$W_f = -W_{mg}$

$W_f = -mgh$

∴ Energy dissipated = mgh

Q.6 (C)

$$\frac{-dU}{dx} = F, \left(\frac{dU}{dx}\right)_{x=0} = 0 \text{ and } \left(\frac{d^2U}{dx^2}\right)_{x=0} = 0$$

Q.7 (A)

$mgH - 2\mu\, mg(d + x) - mgh = 0$

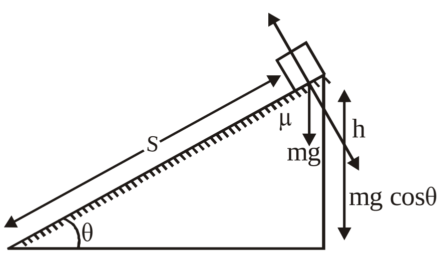
$h = H - 2\mu(d + x)$

Q.8 (B)

$$\frac{1}{2}mv^2 + 0 = 0 + 1$$

$v^2 = 4$ or $v = 2$ m/s

Q.9 (A)



When box is dropped from a height h, then speed at ground is v, therefore using mechanical energy conservation

$$mgh = \frac{1}{2}mv^2 \quad ....(1)$$

when body slides on rough inclined plane, friction force will also act $f = \mu N = \mu\, mg \cos\theta$ Applying work-energy theorem

$$mgh - fs = \frac{1}{2}m\left(\frac{v}{3}\right)^2 - 0$$

$$mgh - f. = \frac{h}{\sin\theta} = \frac{1}{2}m\left(\frac{v}{3}\right)^2$$

$$\left(\sin\theta = \frac{h}{s}\right)$$

$$mgh - \mu mg\cos\theta \times \frac{h}{\sin\theta} = \frac{1}{2}\frac{mv^2}{9} \quad ....(ii)$$

from equation (i) & (ii)

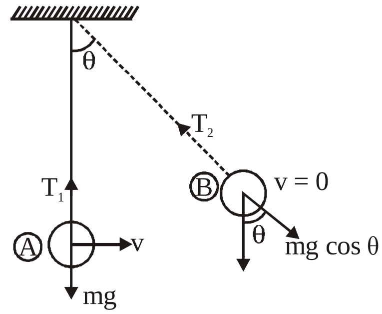
$$mgh\,[1 - \mu\cot\theta] = \left(\frac{1}{9}\right)mgh$$

putting $\theta = 45^o$, $\cot\theta = 1$

$$1 - \mu = \frac{1}{9}$$

$$\Rightarrow \mu = \frac{8}{9}$$

Q.10 (B)



Centripetal force at point A :

$$T_1 - mg = \frac{mv^2}{\ell} \quad ....(1)$$

At point B :

$T_2 = mg\cos\theta$ ....(2)

According to question

$T_1 = 4T_2$ ....(3)

$$\Rightarrow mg + \frac{mv^2}{\ell} = 4\, mg\cos\theta$$ [from equation (1) & (2)]

$$\Rightarrow mg(4\cos\theta - 1) = \frac{mv^2}{\ell} \quad ....(4)$$

According to conservation of energy between point A and B

$$\text{Also } \frac{1}{2}mv^2 + 0 = 0 + mg\ell(1 - \cos\theta)$$

$$mv^2 = 2mg\ell(1 - \cos\theta)$$

$$\frac{mv^2}{\ell} = 2mg(1 - \cos\theta) \quad .....(5)$$

From equation (4) & (5)

$mg\,(4\cos\theta - 1) = 2\,mg\,(1 - \cos\theta)$

$$\Rightarrow 4\cos\theta - 1 = 2 - 2\cos\theta$$

$$\Rightarrow 6\cos\theta = 3$$

$$\Rightarrow \cos\theta = \frac{1}{2}$$

$$\Rightarrow \theta = 60^o$$

Q.11 (C)

$$\Delta K.E. = 0 - \frac{1}{2}mv^2$$

$$\Delta K.E. = -\frac{1}{2}75\,(2)^2$$

Δ K.E. = – 150 J

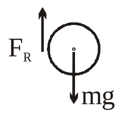
Total work done by forces = –150 J

$-F.\,\Delta x = -150$ J

$F = \dfrac{150}{\Delta x}$ (avg force)

$F = \dfrac{150}{0.25} \Rightarrow F = 600$ N (upward direction)

$F_R - mg = F$



$F_R = F + mg$
$F_R = 600 + 750$
$F_R = 1350$ N
(resistive force by ground)

Q.12 (B)

500 m

$P = \dfrac{mah}{time}$

$\eta = \dfrac{P_{out}}{P_{input}}$

$P_{in} = \dfrac{P_{out}}{\eta}$

$= \dfrac{10^9}{0.5}$

$P_{in} = 2 \times 10^9$

$\dfrac{mah}{time} = 2 \times 10^9$

$m/t = \dfrac{2 \times 10^9}{10 \times 500} = \dfrac{2}{5} \times 10^6$

$= 4 \times 10^5$
$= 400\ m^3$

Q.13 (C)
at t = 0, x = 0.5

$u = \dfrac{x^4}{4} - \dfrac{x^2}{2} \Rightarrow \dfrac{1}{4} \times \dfrac{1}{16} \times \dfrac{1}{4} \times \dfrac{1}{2} \Rightarrow \left|\dfrac{1}{4}\right|$

$\dfrac{du}{dx} = \dfrac{4x^3}{4} - \dfrac{2x}{2} = x^3 - x$

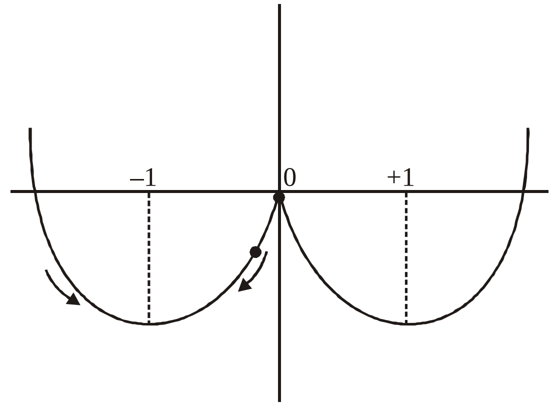
$\dfrac{du}{dx} = x\left(x^2 - 1\right)$

$\dfrac{du}{dx} = 0$ at point of maxima & minima

$x = 0; \quad x = \pm 1$

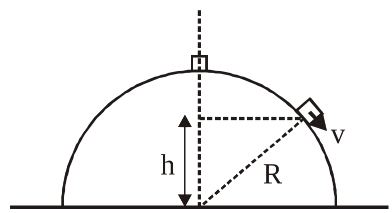
$\left(\dfrac{d^2u}{dx^2}\right)_{x=0} = -1$ point of maxima

$\left(\dfrac{d^2u}{dx^2}\right)_{x=\pm 1} = 2$ point of minima



particle will found between (–1,0)

Q.14 **(A)**



From work energy theorem

$mg\left(R - h\right) = \dfrac{1}{2} mv^2$

$v = \sqrt{2g\left(R - h\right)}$

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Q.1 (3)

$T_{max} = mg + \dfrac{mv^2}{\ell}$

& $T_{min} = \dfrac{m}{\ell}(v^2 - 4g\ell) - mg$

$\therefore \quad \dfrac{5}{1} = \dfrac{g + \dfrac{v^2}{\ell}}{\left(\dfrac{v^2}{\ell} - 5g\right)}$

$\therefore \quad \dfrac{5v^2}{\ell} - 25g = g + \dfrac{v^2}{\ell}$

$\therefore \quad \dfrac{4v^2}{\ell} = 26g$

$v^2 = \frac{13}{2} g \ell$

$v^2_{min} = (5g\ell / 2)$

Q.2 (2)

Given, m = 0.5 kg and u = 20 m/s

Initial kinetic energy (ki) = $\frac{1}{2}$ mu^2

= $\frac{1}{2}$ × 0.5 × 20 × 20 = 100 J

After deflection it moves with 5% of k_i

$\therefore k_f = \frac{5}{100} \times k_i \Rightarrow \frac{5}{100} \times 100$

$\Rightarrow k_f = 5$ J

Now, let the final speed be 'v' m/s, then :

$k_f = 5 = \frac{1}{2} mv^2$

$\Rightarrow$ v2 = 20

$\Rightarrow v = \sqrt{20}$ = 4.47 m/s

Q.3 (2)

P = C

FV = C

$M\frac{dV}{dt}V = C$

$\frac{V^2}{2} \propto t$

$V \propto t^{1/2}$

$\frac{dx}{dt} \propto t^{1/2}$

$x \propto t^{3/2}$

Q.4 (10)

Using work energy theorem,

Wg = ΔK.E.

(10) (g) (5) = $\frac{1}{2}$ (10)v2 – 0

v = 10 m/s

Q.5 (1)

$U = \frac{C}{r}$

$F = -\frac{dU}{dr} = -\frac{C}{r^2}$

$|F| = \frac{mv^2}{r}$

$\frac{C}{r^2} = \frac{mv^2}{r}$

$v^2 \propto \frac{1}{r}$

Q.6 (6)

Let's say the compression in the spring by : y.

So, by work energy theorem we have

$\Rightarrow \frac{1}{2}mv^2 = \frac{1}{2}ky^2$

$\Rightarrow y = \sqrt{\frac{m}{k}} \cdot v$

$\Rightarrow y = \sqrt{\frac{4}{100}} \times 10$

$\Rightarrow$ y = 2m

$\Rightarrow$ final length of spring

= 8 – 2 = 6m

Q.7 (2)

Q.8 [450]

Q.9 (2)

Q.10 (4)

Q.11 (1)

Work done = Change in kinetic energy

$W_{mg} + W_{air\text{-}friction} = \frac{1}{2}m\left(.8\sqrt{gh}\right)^2 - \frac{1}{2}m(0)^2$

$W_{air\text{-}friction} = \frac{.64}{2}$ mgh – mgh = –0.68mgh

Option (1)

Q.12 [400]

Q.13 (1)

Q.14 [40]

Q.15 [16]

Work = ΔK.E.

$W_{friction} + W_{Spring} = 0 - \frac{1}{2}mv^2$

$-\frac{90}{100}\left(\frac{1}{2}mv^2\right) + W_{Spring} = -\frac{1}{2}mv^2$

$W_{Spring} = -\frac{10}{100} \times \frac{1}{2}mv^2$

$$-\frac{1}{2}kx^2 = -\frac{1}{20}mv^2$$

$$\Rightarrow k = \frac{40000\times(20)^2}{10\times(1)^2} = 16\times10^5$$

JEE-ADVANCED PREVIOUS YEAR'S

Q.1 **(D)**

suppose x = r cosθ

y = r sinθ

force on particle is $\frac{K}{r^3}\left(r\cos\theta\hat{i} + r\sin\theta\hat{j}\right)$

force is in radial direction so work done by this force along given path (circle) is zero.

Q.2 **[5]**

E = P.t = 0.5W × 5s = 2.5 J

$= \frac{1}{2}mv^2 \Rightarrow v = 5$ m/s

Q.3 **[5]**

$W_F + W_g = K_f - K_i$

$18\times5 + 1g(-4) = K_f$

$90 - 40 = K_f$

$K_f = 50J = 5\times10J$

Q.4 [0.75]

$F = \left(\alpha y\hat{i} + 2\alpha x\hat{j}\right)$

$W_{AB} = \left(-1\hat{i}.\right).\left(1\hat{i}.\right) = -1J$

$$\begin{bmatrix}\vec{F} = -1i + 2\alpha x\hat{j}\\ \vec{S} = 1\hat{i}\end{bmatrix}$$

Similarly,

$W_{BC} = 1J$

$W_{CD} = 0.25$ J

$W_{DE} = 0.5$ J

$W_{EF} = W_{FA} = 0$ J

∴ New work in cycle = 0.75 J

Q.5 [A, D]

By the energy conservation (ME) between bottom point and point Y

$$\frac{1}{2}mv_0^2 = mgh + \frac{1}{2}mv_1^2$$

$\therefore v_1^2 = v_0^2 - 2gh$ ......(i)

Now at point Y the centripetal force provided by the component of mg

$$\therefore mg\sin30^\circ = \frac{mv_1^2}{R}$$

$$\therefore v_1^2 = \frac{gR}{2}$$

∴ from (i)

$$\frac{gR}{2} = v_0^2 - 2gh$$

At point x and z of circular path, the points are at same height but less then h. So the velocity more than a point y.

So required centripetal $= \frac{mv^2}{r}$ is more.

Center of Mass

ELEMENTARY

Q.1 (4)
self explainatory

Q.2 (2)
Centre of mass is nearer to heavier mass

Q.3 (3)
r = 1.27 Å

$$r_{cm} \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$$

Since centre of mass cannot go beyond bond length

$$r_{cm} \frac{0 + 35.5 \times 1.27}{35.5 + 10} = \frac{35.5 \times 1.27}{36.5} = 1.24 \text{ Å}$$

Q.4 (3)

Q.5 (2)

Q.6 (2)

Q.7 (2)
Q.8 (4)
Q.9 (1)
Body at rest may possess potential energy.

Q.10 (4)

$$a_{cm} = \frac{m_1 g + m_2 g}{m_1 + m_2} = g$$

Q.11 (1)
vector sum of internal forces on system is zero.

Q.12 (2)

Q.13 (1)

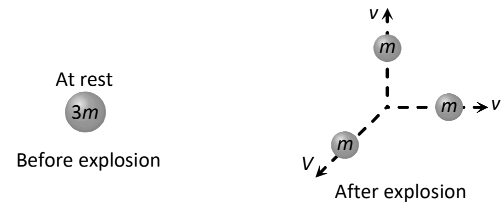
Q.14 (3)

$P = \sqrt{2mE}$

$\therefore\ P \propto \sqrt{m}$ (if E = const.)

$$\therefore \quad \frac{P_1}{P_2} = \sqrt{\frac{m_1}{m_2}}$$

Q.15 (2)
Q.16 (3)



Initial momentum of 3m mass = 0 …(i)
Due to explosion this mass splits into three fragments of equal masses.

Final momentum of system = $m\vec{V} + mv\hat{i} + mv\hat{j}$ …(ii)

By the law of conservation of linear momentum

$$m\vec{V} + mv\hat{i} + mv\hat{j} = 0 \Rightarrow \vec{V} = -v(\hat{i} + \hat{j})$$

Q.17 (4)
Q.18 (1)
Area of F-t curve = A = Impulse.
Impulse = dP = A = mv – 0

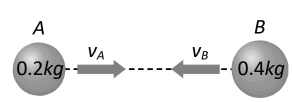
$$\therefore v = \frac{A}{M}.$$

Q.19 (1)

Q.20 (1)
If mass = m
first ball will stop
$\Rightarrow v = 0$
so: K.E. = 0 (min)
In other cases there will be some kinetic energy
(K.E. can't be negative)

Q.21 (3)
According to law of conservation of linear momentum both pieces should possess equal momentum after explosion. As their masses are equal therefore they will possess equal speed in opposite direction.

Q.22 (3)



Initial linear momentum of system = $m_A \vec{v}_A + m_B \vec{v}_B$
$= 0.2 \times 0.3 + 0.4 \times v_B$
Finally both balls come to rest
$\therefore$ final linear momentum = 0
By the law of conservation of linear momenum
$0.2 \times 0.3 + 0.4 \times v_B = 0$

$$\therefore \quad v_B = -\frac{0.2 \times 0.3}{0.4} = -0.15 \text{ m/s}$$

Q.23 (1)

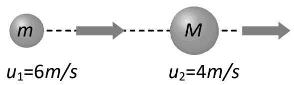
$$v_1 = \frac{(m_1 - em_2)u_1}{m_1 + m_2} + \frac{m_2(1+e)u_2}{m_1 + m_2}$$

$$= \frac{(m - e2m)u_1}{m + 2m} + \frac{2m(1+e)\times 0}{m+2m} = 0$$

$\Rightarrow$ $0 = m - e2m$

$\Rightarrow$ $e = 1/2$

Q.24 (1)



$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2u_2}{m_1 + m_2}$$

Substituting $m_1 = 0$, $v_1 = -u_1 + 2u_2$

$\Rightarrow$ $v_1 = -6 + 2(4) = 2$ m/s

i.e. the lighter particle will move in original direction with the speed of 2 m/s.

Q.25 (2)

Impulse = change in momentum

$mv_2 - mv_1 = 0.1 \times 40 - 0.1 \times (-30)$

Q.26 (4)

By the conservation of momentum

$40 \times 10 + (40) \times (-7) = 80 \times v \Rightarrow v = 1.5$ m/s

Q.27 (4)

Due to elastic collision of bodies having equal mass, their velocities get interchanged.

Q.28 (3)

Initial momentum of the system = $mv - mv = 0$

As body sticks together

$\therefore$ final momentum = 2mV

By conservation of momentum 2mV = 0

$\therefore$ $V = 0$

Q.29 (2)

By momentum conservation before and after collision.

$m_1V + m_2 \times 0 = (m_1 + m_2)v$

$$\Rightarrow \quad v = \frac{m_1}{m_1 + m_2}V$$

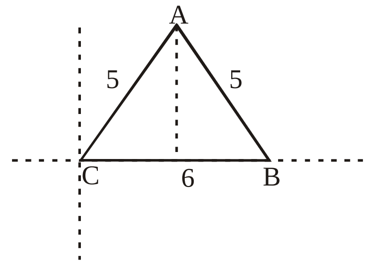
i.e. Velocity of system is less than V.

JEE-MAIN
OBJECTIVE QUESTION

Q.1 (4)

Centre of mass is a point which can lie within or outside the body.

Q.2 (1)



$$y_{cm} = \frac{0 \times (2\rho) + 2 \times 5\rho \times 2}{2(\rho)6 + 2 \times 5 \times \rho} = \frac{10}{11}$$

$$4 - \frac{10}{11} = \frac{34}{11}$$

Q.3 (4)

$A_1 = 2r \times r = 2r^2$

$$A_2 = \frac{\pi r^2}{2}$$

$$x_1 = \frac{r}{2}$$

$$x_2 = \frac{4r}{3\pi}$$

$$x_{cm} = \frac{2r^2 \times \frac{r}{2} - \frac{\pi r^2}{2} \times \frac{4r}{3\pi}}{2r^2 - \frac{\pi r^2}{2}} = \frac{r^3\left[1 - \frac{2}{3}\right]}{r^2\left[\frac{4-\pi}{2}\right]} = \frac{2r}{3[4-\pi]}$$

Q.4 (4)

centre of mass is at a height of h/4 from base.

Q.5 (3)

Q.6 (3)

Centre of mass of two particle system lies on the line joining the two particles

Q.7 (3)

Q.8 (2)

$y_{cm} = 0$

$$\frac{1}{8} \times 0.14 + \frac{7}{8} \times h = 0$$

$$\therefore \frac{7h}{8} = -\frac{0.14}{8} \Rightarrow h = -0.02 \text{ below x-axis.}$$

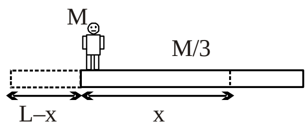
Q.9 (2)

Let x be the displacement of man. Then displacement of plank is L – x.

For centre of mass to remain stationary

$\frac{M}{3}(L - x) = M \cdot x$

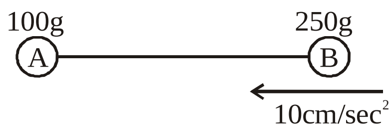
$\Rightarrow \quad x = \frac{L}{4}$



Q.10 (1)

$\vec{F}_{net} = 0$

so $\vec{a}_{com} = 0$



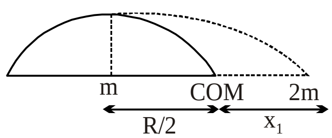
$m_1\vec{a}_1 + m_2\vec{a}_2 = 0$

$100 \times a_1 + 250(-10) = 0$

$a_1 = 25$ cm/sec^2 east

Q.11 (3)

Centre of mass hits the ground at the position where original projectile would have landed.



$\frac{m.R}{2} = 2mx_1 \Rightarrow x_1 = \frac{R}{4}$

$\therefore$ Distance $= R + \frac{R}{4} = \frac{5R}{4}$

Q.12 (1)

$v_{cm} = \frac{1 \times 2 + \frac{1}{2} \times 6}{1 + 1/2} = \frac{10}{3}$ m/sec.

Q.13 (4)

$v_{cm} = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}$

$\therefore v_{cm} = \frac{m(2\hat{i}) + m(2\hat{j})}{2m}$

$a_{cm} = \frac{m(i + j) + m(0)}{2m}$.

v_{cm} has same direction as of a_{cm}

$\therefore$ straight line.

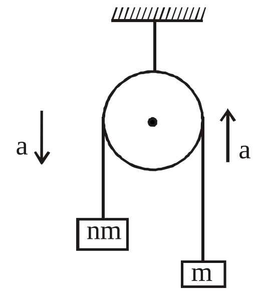
Q.14 (3)

$a = \frac{(nm - m)}{nm + m} g$

$= \frac{(n-1)}{(n+1)} g$

$a_1 = a_2 = a$

$a_{cm} = \frac{nma_1 - ma_2}{(nm + m)} = \frac{(n-1)}{(n+1)} \times a$



$a_{cm} = \frac{(n-1)^2}{(n+1)^2} g$.

Q.15 (4)

Q.16 (2)

Q.17 (2)

Q.18 (1)

$v_{cm} = 0$

$mv_B + m(v_B + v_{rel}) = 0$

$\therefore v_B = -\frac{mv_{rel}}{m + M}$

– sign means baloon moves downward

Q.19 (3)

Centre of mass will not move in horizontal direction.

Let x be the displacement of boat.

$80(8 - x) = 200x$

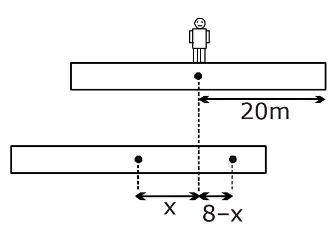
$640 - 80x = 200x$

$x = 2.3$ m

Now, Required

distance from the shore.

$= 20 - (8 - x)$



$= 20 - (8 - 2.3)$

$= 20 - 5.7$

$= 14.3$ m

Q.20 (3)
C_1 will move but C_2 will be stationary with respect to the ground.

Q.21 (2)
Velocity become double.

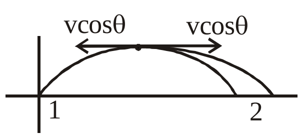
Q.22 (1)
$500 \times 10 = 550 \times v$

$v = \frac{500}{55} = \frac{100}{11} m/s$.

Q.23 (2)
$V_{com} = V \cos\theta$

$V \cos\theta = \frac{-m0 + mv_2}{2m}$

$\therefore v_2 = 2V \cos\theta$



Q.24 (4)
Speed is constant so K.E. → Constant
Gravitational potential energy change.
∵ Momentum = $m\vec{v}$
∵ Direction of $\vec{v}$ changes
∴ Momentum changes

Q.25 (4)

$\frac{P^2}{2m} = K.E.$

$\ln \frac{P^2}{2m} = \ln K.E.$

$2\ln P - \ln(2m) = \ln K.E.$
So the graph between lnp & lnk is straight line with intercept.

Q.26 (2)
Here net force = 0
means momentum is conserved.
$p_i = p_f$
$0 = \vec{p}_1 + \vec{p}_2 \Rightarrow \vec{p}_1 = -\vec{p}_2$

$K.E. = \frac{p^2}{2m} \Rightarrow \therefore \frac{K_1}{K_2} = \frac{m_2}{m_1}$

Q.27 (1)
According to Newton's second law of motion.

$\vec{F} = \frac{d\vec{p}}{dt}$

If $\vec{F}_{net} = 0$

then $\vec{p}$ = conserved

Q.28 (3)
$P_i = mv_1 + mv_2$
$P_f = (m + M) v$

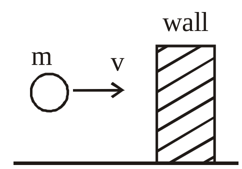
$P_i = P_f \Rightarrow v = \frac{mv_1 + Mv_2}{(m + M)}$
By energy consarvation

$\frac{1}{2} mv_1^2 + \frac{1}{2} Mv_2^2 = \frac{1}{2} (M + m) v^2 + \frac{1}{2} kx^2$

$\Rightarrow mv_1^2 + Mv_2^2 = (M + m) \frac{(mv_1 + Mv_2)^2}{(M+m)^2} + kx^2$

solving $x = (v_1 - v_2) \sqrt{\frac{mM}{(M+m)k}}$.

Q.29 (1)



Initial momentum of body = mv
& final momentum of body = – mv
Change in momentum = 2mv

Q.30 (3)

$\vec{F}_{net} = 0$

then $\vec{p}$ = conserved

$\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = 0$

$\vec{p}_3 = -(\vec{p}_1 + \vec{p}_2)$

$m\vec{v}_3 = -m(\vec{v}_1 + \vec{v}_2)$

$\therefore \vec{v}_3 = -\left[\left(3\hat{i} + 2\hat{j}\right) + \left(-\hat{i} - 4\hat{j}\right)\right]$

$\vec{v}_3 = -2\hat{i} + 2\hat{j}$

Q.31 (1)

$\vec{F}_{net} = 0$

then $\vec{p}$ = conserved

$p_i = p_f$
$m_1 v = m_2(0) + (m_1 - m_2) v_1$

$v_1 = \frac{m_1 v}{(m_1 - m_2)}$

Q.32 (1)

As $f_{net} = 0$ from momentum conservation

$$(A-4)v_1 = 4v \Rightarrow v_1 = \frac{4v}{(A-4)}$$

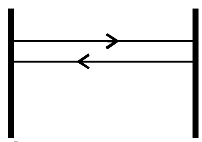
Q.33 (A) [2] (B) [3]

(1) It could be non-zero, but it must be constant.

(2) It could be non-zero and it might not be constant.

Q.34 (2)

Total travelled distance = 2d



then

Time between two collisions = $\frac{2d}{v_0}$

So no. of collision/sec = $\frac{v_0}{2d}$

Impulse in one collision = $mv_0 - (-mv_0) = 2mv_0$

$$F = 2mv_0 \times \frac{v_0}{2d} = \frac{mv_0^2}{d}$$

Q.35 (2)

$$v_1 = \sqrt{2gh} = \sqrt{2\times10\times10} = 10\sqrt{2}$$

$$k_2 = \frac{1}{4}k_1 \Rightarrow v_2{}^2 = \frac{1}{4}v_1{}^2$$

$$\therefore v_2 = \frac{v_1}{2} = 5\sqrt{2}$$

$|\Delta P| = |-mv_2 - (mv_1)| = m\,|-v_2 - v_1|$

$$|\Delta P| = 50\times10^{-3}\times\frac{3}{2}\times10\sqrt{2} = \frac{15\times10^{-1}}{\sqrt{2}}$$

$J = \Delta P = 1.05$ N-s.

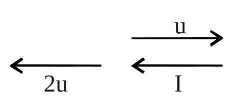
Q.36 (2)

Impulse = change in momentum

$-I = -m2u - mu$

$I = 3mu$

W.D. = change in K.E.



$$\text{W.D.} = \frac{1}{2}m(2u)^2 - \frac{1}{2}mu^2$$

$$= \frac{3}{2}mu^2 \Rightarrow \text{W.D.} = \frac{Iu}{2}$$

Q.37 (3)

Impulse = change in momentum

$$\int F.dt = \Delta P$$

Given $\int F.dt = J$

Now, Contact time is twice than the earlier.

$$\int \vec{F}.2dt = J' \Rightarrow J' = 2J$$

Q.38 (2)

$mv_i + mvj + 2mv_3 = 0$

$$\vec{v}_3 = -\frac{(vi + v\hat{j})}{2} = -\frac{v}{2}(i+\hat{j}) = -\frac{v}{\sqrt{2}}.$$

$$k_f = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}2m\frac{v^2}{2}.$$

$$k_f = \frac{3mv^2}{2}.$$

Q.39 (3)

From momentum conservation

$mu = 2mv$

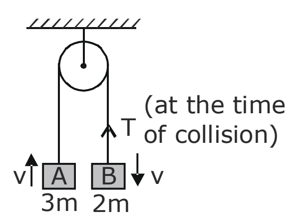
$$\Rightarrow v = \frac{u}{2}$$

from energy conservation

$$\frac{1}{2}\times2m\times\left(\frac{u}{2}\right)^2 = 2\,mgh$$

$$\Rightarrow h = \frac{u^2}{8g}$$

Q.40 (4)



Impulse = change in momentum

So, $-T\Delta t = 2mv - mu$ (for bullet)

$I = T\Delta t = 3mv$ (for mass 3m)

$3mv = 2mv - mu$

$$v = u/5 \Rightarrow I = \frac{3mu}{5}$$

Q.41 (2)

If e = 1 then angle = 45°

If 0 < e < 1 then angle is less than 45° with the horizontal. So 30° is not possible.

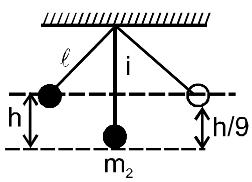
Q.42 (1)
In inelastic collision, due to collision some fraction of mechanical energy is retained in form of deformation potential energy.
$\therefore$ thus K.E. of particle is not conserved.
In absence of external forces momentum is conserved.

Q.43 (4)
$0.5 \times v_p + m \times 0 = 5.05\, v$

$\therefore \frac{v_f}{v_i} = \frac{0.05}{5} = 10^{-2}$

$\Rightarrow \frac{\frac{1}{2}m(v_f)^2}{\frac{1}{2}m(v_i)^2} = (10^{-2})^2 = 10^{-4}.$

Q.44 (1)
$m_1\sqrt{2gh} + 0 = (m_1 + m_2)\, v$



$v = \frac{m_1\sqrt{2gh}}{(m_1 + m_2)}$

$\therefore v^2 - u^2 + 2g \times \frac{h}{9} = 6 + 2g \times \frac{h}{4} = \frac{gh}{2}$

$\therefore v = \sqrt{\frac{gh}{2}}$

Also, $\sqrt{\frac{gh}{2}} = \frac{m\sqrt{2gh}}{m_1 + m_2} \Rightarrow 2m_1 + m_1 + m_2;$

$\therefore \frac{m_1}{m_2} = 1.$

Q.45 (2)
by conservation of linear momentum
$P_i = P_f$
$\Rightarrow$ mv = (100 m) u
$\Rightarrow$ u = v/100
$P_i = P_f$
$\Rightarrow$ mv = (100 m) u $\Rightarrow$ u = v/100

Q.46 (3)
$\because e = 1$
As collision is elastic therefore $v_i = v_f$

So $\Delta K = 0 \Rightarrow k_f = k_i = \frac{1}{2}m\left(u_1^2 + u_2^2\right)$

Q.47 (3)
In absence of external force. Momentum of the system is conserved.

Q.48 (3)
If e = 1 and $m_1 = m_2$ then after collision velocity interchange

Q.49 (2)
from energy conservation

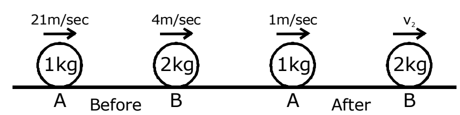
$mql = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gl}$

from momentum conservation

$m\sqrt{2gl} = mv' \Rightarrow v' = \sqrt{2gl}$

$KE = \frac{1}{2}m \times 2gl = mgl$

Q.50 (2)



$21 \times 1 - 4 \times 2 = 1 + 2v_2$

$21 - 8 = 1 + 2v_2$

$2v_2 = 12 \Rightarrow v_2 = 6\text{m/sec}$

$e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{6-1}{21+4} = \frac{5}{25} = \frac{1}{5}$

$e = 0.2$

Q.51 (3)

$M_A = \rho \times \frac{4}{3}\pi r^3$ $\quad e = \frac{1}{2}$

$M_B = \rho \times \frac{4}{3}\pi (2r)^3 = 8M_A$

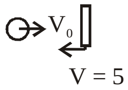
$m_A v + 0 = m_A v_1 + m_B v_2$ ....(i)
$ev = v_2 - v_1$ ....(ii)

Adding (i) + (ii) $= 9v_2 = v + \frac{v}{2} = \frac{3v}{2}$

$\therefore v_1 = v_2 - \frac{v}{2} = \frac{v}{6} - \frac{v}{2} = -\frac{v}{3}.$

$\therefore \frac{v_1}{v_2} = \frac{v/3}{v/6} = 2.$

Q.52 (1)



$V_2 = Z_0$

Vel. of Sep = Vel of approach ($\therefore$ elastic)

$\therefore\ 20 + 5 = V - 5$

$\Rightarrow V = 30$ m/s **Ans.**

$v_b = -(v_0 + 2v)$ $\qquad \therefore m_1 >> m_2$

$v_b = -(20 + 10) = -30$ m/sec.

Q.53 (2)

Q.54 (1)

$mu = mv_1 + mv_2$ ....(i)

$u = v_1 + v_2$ ....(i)

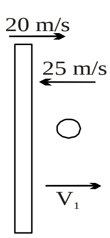
$\dfrac{v_2 - v_1}{u} = e$ ......(ii)

as solving have $\dfrac{v_1}{v_2} = \left(\dfrac{1-e}{1+e}\right)$.

Q.55 (1)

Let v_1 is the velocity of wall after collision.

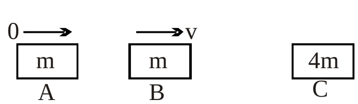
$$e = \frac{V_1 - 20}{20 - (-25)} \ (e = 1)$$



$v_1 = 65$ m/s

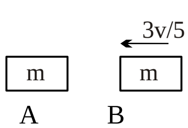
Q.56 (1)

1^{st} Collision



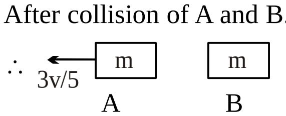
2^{nd} Collision

Velocity of B $v = \dfrac{mv + 4m(0 - v)}{5m} = \dfrac{3m}{5}$



After collision of A and B.

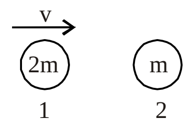
$\therefore$



Q.57 (2)

Let mass of ball 2 is m and mass of ball 1 is 2 m.

$$v_1 = \frac{m_1u_1 + m_2u_2 + m_2e(u_2 - u_1)}{m_1 + m_2}$$

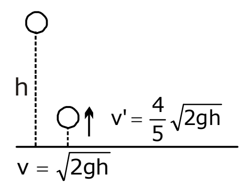


$\dfrac{v}{3} = \dfrac{2mv + em(0 - v)}{3m} \Rightarrow e = 1$

So elastic collision.

Q.58 (3)

Just before collision, speed of ball $v = \sqrt{2gh}$ and just after collision $v' = \dfrac{80}{100}\sqrt{2gh} = \dfrac{4}{5}\sqrt{2gh}$



$v^2 - u^2 = 2aS$

Let h' is the maximum height after collision.

$$0 - \left(\frac{4}{5}\sqrt{2gh}\right)^2 = 2x(-g) \times h'$$

$$\frac{16}{25} \times 2gh = 2gh$$

$$h' = \frac{16}{25}h$$

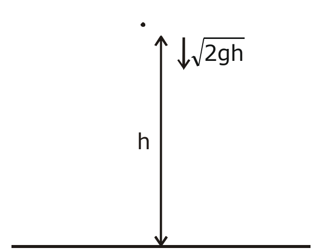
Q.59 **(1)**

From energy conservation

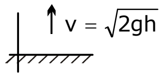
$$\frac{1}{2}m\left(\sqrt{2gh}\right)^2 + mgh = \frac{1}{2}mv^2$$

$$v = 2\sqrt{gh}$$

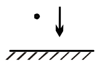
$$e = \frac{\sqrt{2gh}}{2\sqrt{gh}}$$



$\therefore\ e = \dfrac{1}{\sqrt{2}}$,



Q.60 (3)



$\sqrt{2\times10\times5} = 10$ m/sec.

$$\therefore\ \frac{10}{10} + \frac{2\times e\times10}{10} + \frac{2\times e^2\times10}{10} + ...$$

$1 + 2\,[e + e^2 + ...]$

$$1 + \frac{2e}{1-e} = 3 \text{ sec.}$$

Q.61 (1)

$v = Av_1 + v_2$ (1)

$$1 = \frac{v_1 - v_2}{v} \Rightarrow v = v_1 = v_2$$

$$v_1 = \frac{2v}{A+1} \qquad v_2 = v\left(\frac{1-A}{1+A}\right)$$

Q.62 (3)

$$5\times10 = \frac{5}{2}(0) + \frac{5}{2}(v_1) \Rightarrow v_1 = 20 \text{ m/sec}$$

$$KE = \frac{1}{2}\times\frac{5}{2}(20)^2 - \frac{1}{2}\times5(10)^2$$

$= 500 - 250 = 250$ J.

Q.63 (2)

$$E_i = \frac{1}{2}mu_1^2 + \frac{1}{2}mu_2^2$$

$$m(u_1 - u_2) = 2\,mu \Rightarrow u = \frac{u_1 - u_2}{2}$$

$$\text{Energy loss} = \frac{1}{2}\times\frac{2m}{4}(u_1 - u_2)^2 - \frac{1}{2}m(u_1^2 + u_2^2)$$

Q.64 (4)

$mu = nmu_1 + 1mu_2$

$$1 = \frac{u_1 - u_2}{u}$$

$u = n(u+u_2) + u_2$

$u = nu + nu_2 + u_2$ ....(1)

$u = nu_1 + u_1\text{-}u$

$2u = (n+1)\,u_1$ ....(2)

$$\frac{\frac{1}{2}nmu_1^2}{\frac{1}{2}mu^2} = \frac{n\frac{4u^2}{(n+1)^2}}{u^2} = \frac{4n}{(n+1)^2}$$

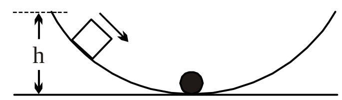
Q.65 (4)

$\Delta p = 0.1\ (6+4)$

$= 0.1\times10 = 1$ NS

Q.66 (1)

$$mgh = \frac{1}{2}mv^2$$



$v = \sqrt{2gh}$

By momentum conservation

$m\sqrt{2gh} + 0 = 2mv'$

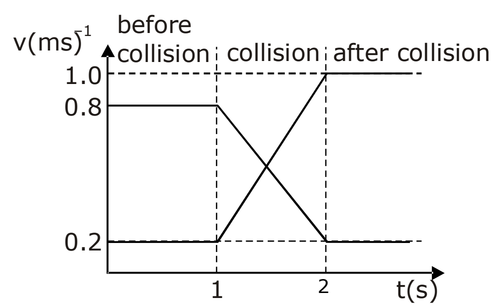
$$v' = \frac{\sqrt{2gh}}{2}$$

By energy conservation

$$\frac{1}{2}(2m)v'^2 = 2mgh',\ m\frac{(2gh)}{4} = 2mgh'$$

$$h' = \frac{h}{4}$$

Q.67 (4)



(i) $\because$ v is +ve for both.

(ii) Yes (when maximum compression)

(iii) $\because$ S have greater velocity after collision then R have before collision and K.E. of S will be less then initial K.E. of R

$$\frac{1}{2}m_sV_s^2 < \frac{1}{2}m_R(V_R)^2$$

but $V_S > V_R$ So $m_s < m_R$

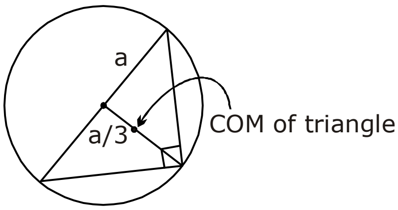
JEE-ADVANCED
OBJECTIVE QUESTIONS

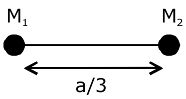
Q.1 (B)

COM can lie anywhere within the radius r.

Q.2 (C)

COM of circle is at O. Let M_1 is mass of circle and M_2 is mass of triangle



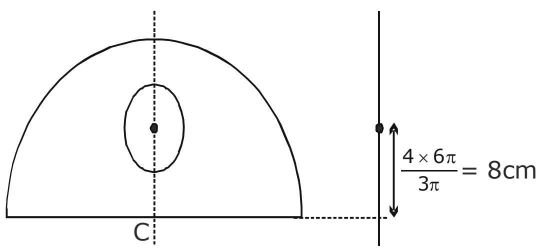


Distance of COM from centre of circle

$$r_1 = \frac{M_2\ell}{M_1+M_2} = \frac{-\sigma a^2}{\sigma\pi a^2 - \sigma a^2} \times \frac{a}{3}$$

$$= \frac{a^2 \times a}{3a^2(\pi-1)} = \frac{a}{3(\pi-1)}$$

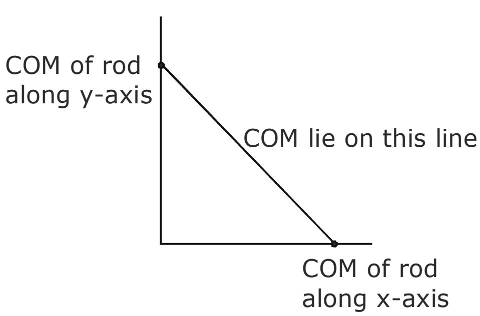
Q.3 (B)



COM of semic circular disc = $\frac{4R}{3\pi}$

So from point C distance of COM is 8 cm.
Center of mass coincides

Q.4 (D)



and equation of line is $\frac{x}{L}+\frac{y}{L}=1$

Q.5 (D)
Acceleration of COM does not depend on position of force.

Q.6 (B)
Since no external force acting on system hence V_{CM} remain constant.

Q.7 (D)
An external force of $3m\omega^2R$ is required which can act anywhere on system.

Q.8 (C)

Centre of mass of uniform semi-circular disc is at $\frac{4R}{3\pi}$

Centre of mass of uniform semi-circular ring is at $\frac{2R}{\pi}$

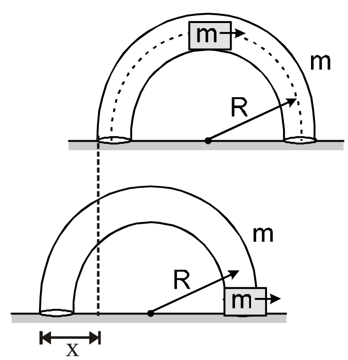
Centre of mass of solid hemi-sphere is at $\frac{3R}{8}$

Centre of mass of hemi-sphere shell is at $\frac{R}{2}$

C	T	H	R	S	D
h	h	R	2R	3R	4R
4	3	2	π	8	3π

Q.9 (C)
Since there is no ext. force on system

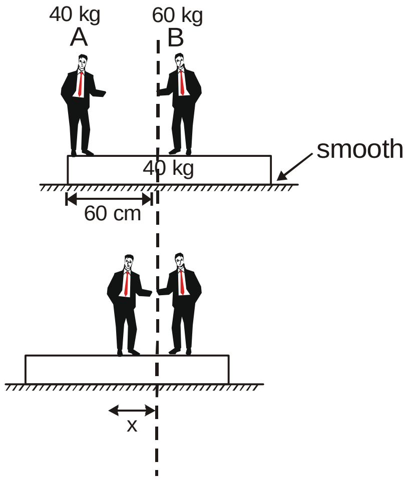
$m(R-x)+m(-x)=0$
$x = R/2.$



Alternate : Let the tube displaced by x towards left, then ;

$$mx = m(R-x) \Rightarrow x = \frac{R}{2}$$

Q.10 (C)
Taking the origin at the centre of the plank.



$m_1\Delta x_1 + m_2\Delta x_2 + m_3\Delta x_3 = 0$

$(\because\ \Delta x_{CM} = 0)$

(Assuming the centres of the two men are exactly at the axis shown.)

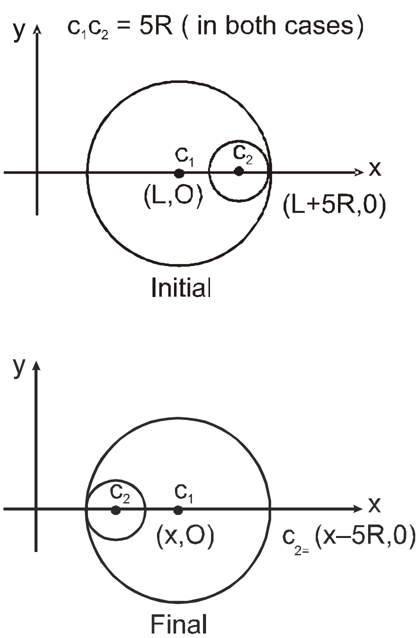
$60(0) + 40(60) + 40(-x) = 0$, x is the displacement of the block.

$\Rightarrow$ $x = 60$ cm

i.e. A & B meet at the right end of the plank.

Q.11 (B)

Since all the surfaces are smooth, no external force is acting on the system in horizontal direction. Therefore, the centre of mass of the system in horizontal direction remains stationary.



x-coordinate of COM initially will be given be given by–

$$x_i = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{(4M)(L) + M(L + 5R)}{4M + M} = (L + R)....(1)$$

Let (x,0) be the coordinates of the centre of large sphere in final position. Then x-coordinate of COM finally will be

$$x_f = \frac{(4M)(x) + M(x - 5R)}{4M + M} = (x - R).........(2)$$

Equating (1) and (2), we have

$$x = L + 2R$$

Therefore, coordinates of large sphere, when the smaller sphere reaches the other extreme position, are

$(L + 2R, 0)$ **Ans**

Q.12 (A)

$y_{CM} = 0$

$$y_{CM} = \frac{m}{4}y_1 + \frac{3m}{4}y_2 \qquad y_1 = +15$$

$\therefore$ $y_2 = -5$ cm

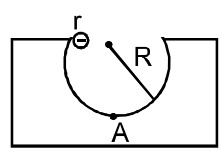
Q.13 (B)

when ball reaches pt A.

then block get shifted by x

$\therefore$ but since than there is no ext

force therefore com remain at its position



$[(R-r) - x]m = Mx$

$$\therefore\ x = \frac{m(R - r)}{M + m}$$

Q.14 (C)

Using momentum conservation

$$MV = mv \quad V = \frac{mv}{M} \text{......(i)}$$

using energy conducts equation

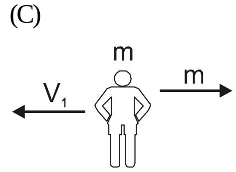
$$mg(R-r) = \frac{1}{2}mv^2 + \frac{1}{2}Mv^2 \text{ ......(ii)}$$

on solving we get $v = m\sqrt{\dfrac{2g(R - r)}{M(m - m)}}$.

Q.15 (A)

$$a_{com} = \frac{\vec{F}_{ext}}{M} = \frac{\overrightarrow{Mg} + \vec{R}}{M} \quad \text{(Rem. } \vec{R} \text{ is vector)}$$

Q.16 (C)



$P_i = 0$ ...(i)
$P_f = MV - mV_1$ ....(ii)

$MV - mV_1 = 0 \Rightarrow u = \frac{M}{m}V.$

using $V_1^2 = u^2 + 2ax.$
$a = \mu g.$

$$\left(\frac{MV}{m}\right)^2 = 0 + 2\mu g\, x.$$

$$\therefore \quad x = \frac{M^2V^2}{2m^2\mu g}$$

Q.17 (C)
use $m_1v_1 = m_2v_2 = P$

$$\text{F.E.} = \frac{1}{2}mv_1^2 + \frac{1}{2}m_2v_2^2$$

$$= \frac{1}{2}m_1\left(\frac{P}{m_1}\right)^2 + \frac{1}{2}m_2\left(\frac{P}{m_2}\right)^2$$

$$= \frac{1}{2}\frac{P^2(m_2 + m_1)}{m_1m_2}.$$

Q.18 (B)
If we treat the train as a ring of mass 'M' then its COM will be at a distance $\frac{2R}{\pi}$ from the centre of the circle.
Velocity of centre of mass is :

$$V_{CM} = R_{CM}\,.\omega$$

$$= \frac{2R}{\pi}.\omega = \frac{2R}{\pi}.\left(\frac{V}{R}\right)\ (\because\ \omega = \frac{V}{R})$$

$$\Rightarrow \quad V_{CM} = \frac{2V}{\pi} \Rightarrow MV_{CM} = \frac{2MV}{\pi}$$

As the linear momentum of any system = MV_{CM}

$\therefore$ The linear momentum of the train = $\frac{2MV}{\pi}$ **Ans.**

Q.19 (A)
Using momentum conservation

$$\vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \vec{p}_4 = 0$$

$$\vec{p}_1 = -\vec{p}_2 - \vec{p}_3 - \vec{p}_4$$

$$|\vec{p}_1| = \sqrt{p_2^2 + p_3^2 + p_4^2}$$

$$\text{K. E}_1 = \frac{p_1^2}{2m} = \frac{p_2^2 + p_3^2 + p_4^2}{2m} = E_0 + E_0 + E_0$$

Total energy = $3E_0 + E_0 + E_0 + E_0 = 6E_0$

Q.20 (A)

$$I = f \times \Delta t \text{ and } F = \frac{m(\sqrt{2gh_2} + \sqrt{2gh_1})}{\Delta t}$$

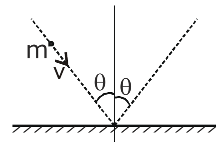
$$F = \frac{100 \times 10^{-3}(\sqrt{2 \times 9.8 \times 0.625} + \sqrt{2 \times 9.8 \times 2.5})}{0.01}$$

$F = 105$ N

Q.21 (B)
(i) From M.C. mv = 2mv'
v' = v/2
(ii) from M.C. mv = 2mv'
v' = v/2
(iii) Impulse = mv = 3mv'

$$v' = \frac{v}{3}$$

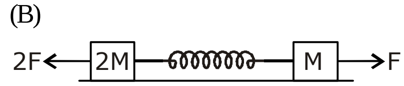
Q.22 (B)
$\Delta P = 2mv\cos\theta$
F_{avg} unit volume
$= (2mv\cos\theta)(nv)$
$= 2mnv^2\cos\theta$



$$\text{Pressure} = \frac{F_\perp}{\text{area}} = 2mnv^2\cos\theta\cos\theta$$

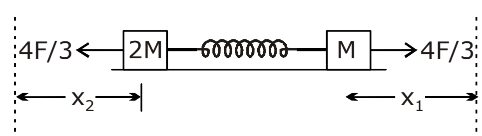
Q.23 (B)
In centre of mass frame total momentum of the system is always zero.
Hence momentum of other particle is $-\vec{p}$.

Q.24 (B)



$$a_{COM} = \frac{F}{3M}$$

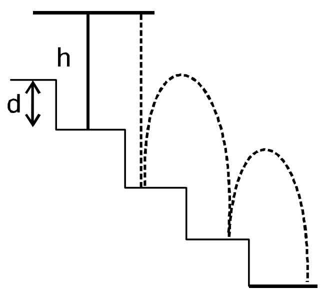
w.r. to COM



$$\frac{4F}{3}x_1 + \frac{4F}{3}x_2 = \frac{1}{2}k(x_1 + x_2)^2$$

$$\frac{8F}{3K} = (x_1 + x_2)$$

Q.25 (C)



Velocity of the ball on striking = $\sqrt{2gh}$

After that ball goes to height less than (h) due to inelastic collission = $\sqrt{2g}(h-d)$.

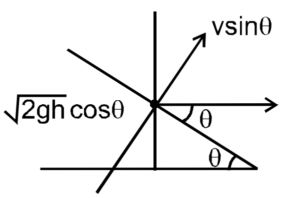
$\therefore \sqrt{2g(h-d)} = e\sqrt{2gh}$

$h - d = e^2 h \Rightarrow \dfrac{h}{d} = \dfrac{1}{1-e^2}$.

Q.26 (A)

$e = \dfrac{v\sin\theta}{\sqrt{2gh}\cos\theta}$

apply conservation of momentum



$m\sqrt{2gh} = m\, v\cos\theta$ ......(i)

$e\sqrt{2gh}\cos\theta \times m = mv\cos\theta$ ......(ii)

$\dfrac{\tan\theta}{e} = \cot\theta$.

$\therefore$ $e = \tan^2\theta$ on solving

Q.27 (A)

Here $e = 1$

if ball rebound elastically

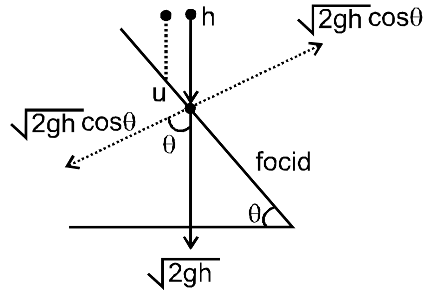
$v_{LOI} = u_{LOI}$

$\therefore$ Along line of impact momentum conservation

$\int Ndt = mv - (-mu) = mv + mu = 2mu$

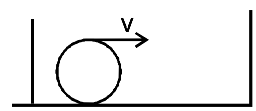
Alog LO I $\qquad u_{LOL} = u\cos\theta$

$= \sqrt{2gh}\cos\theta$



$J = 2m\cos\theta.\ \sqrt{2gh}$.

Q.28 (A)



$v_1 = v$

and $\quad v_2 = ev$ and $t = \dfrac{d}{v_{avg}}$

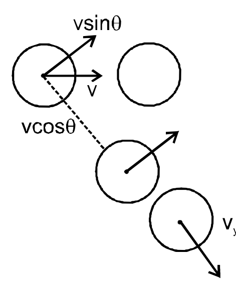
$\langle v_{avg}\rangle = \dfrac{e}{t}$

$\dfrac{2}{3}v = \dfrac{2\ell}{\dfrac{\ell}{v}+\dfrac{\ell}{ev}} \quad \dfrac{1}{3} = \dfrac{e}{e+1}$.

get $\quad e = 0.5$

Q.29 (C)

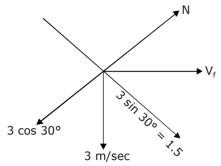
$m_2 v\cos\theta = 3v_y$



$\dfrac{v_y}{v\cos\theta} = \dfrac{2}{3}$

Also $\quad e = \dfrac{v_y}{v\cos\theta} = \dfrac{2}{3}$.

Q.30 (B)



$mV_f \cos 30° = 1.5m$

$V_f \cos 30° = 1.5$

$v_f = \sqrt{3}\ m/s$

Q.31 (D)

$mgh = KE_A + KE_B$

$0.25 \times 0.45 \times 10 = 1 + \frac{1}{2}(0.25)v^2$

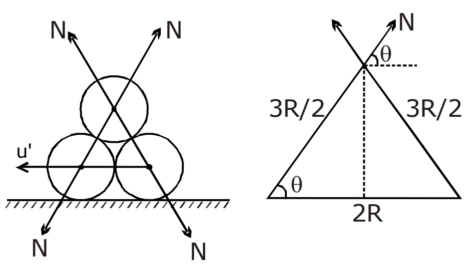
v = 1 m/s

Ball B is heavy so ball A velocity is towards left

Q.32 (C)

$\int 2N \sin\theta .dt = Mv_0$ ....(i)

$\int N\cos\theta . dt = Mu'$



$\int N.dt = \frac{mv_0}{2\sqrt{5}} . 3$

$\sin\theta = \frac{\sqrt{(3/2)R^2 - R^2}}{3/2R}$

$\sin\theta = \frac{\sqrt{5}}{3}$; $\cos\theta = \frac{2}{3}$

$\frac{mv_0 3}{2\sqrt{5}} \frac{2}{3} = mv' \Rightarrow v' = \frac{v_0}{\sqrt{5}}$

Q.33 (C)

Impulse = change in momentum

$\int 2N\sin\theta dt = \frac{mv_0}{2}$ ....(i)

$\int N.\cos\theta dt = mv'$ ....(ii)

from (i) and (ii)

$\int 2N \times \frac{\sqrt{5}}{3} dt = \frac{mv_0}{2}$

$\int \frac{2N}{3} dt = mv'$

On dividing

$\frac{2N \times \sqrt{5}}{3} \times \frac{3}{2N} = \frac{v_0}{2v'}$

$v' = \frac{v_0}{2\sqrt{5}}$

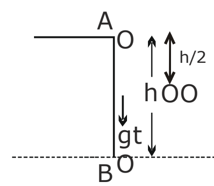
Q.34 (D)

time to reach $\frac{h}{2}$ from top by A

$t = \sqrt{\frac{h}{g}}$

for body B

$\frac{h}{2} = v\sqrt{\frac{h}{g}} - \frac{1}{2}g\left(\sqrt{\frac{h}{g}}\right)^2$



$h = v\sqrt{\frac{h}{g}}$, $v = \sqrt{hg}$

velocity of body B at $\frac{h}{2}$

$v_f = \sqrt{hg} - g\sqrt{\frac{h}{g}}$

$v_f = 0$

Now

momentum conservation

mg.t = 3mv'

gt/3 = v'

Energy conservation

$\Rightarrow \frac{1}{2} 3m (gt/3)^2 + 3mg\frac{h}{2} = \frac{1}{2} 3m.v_1^2$

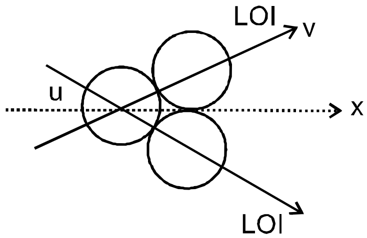
$v_1 = \frac{\sqrt{10gh}}{3}$

Q.35 (A)

In x direction : Applying conservation of momentum

mu = 2mvcos30

$$v = \frac{u}{2\cos 30^\circ} = \frac{u}{\sqrt{3}}.$$

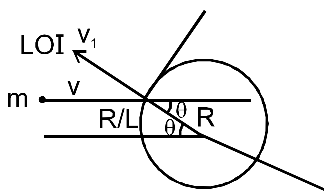


Also $e = \frac{v}{u\cos 30^\circ} = \frac{u}{\sqrt{3}u} \times \frac{2}{\sqrt{3}}.$

$\therefore e = \frac{2}{3}.$

Q.36 (C)

$\sin\theta = \frac{R/2}{R}$; $\theta = 30^\circ$



Both have equal mass it means along LOI particle transfer it velocity to disc which is $v\cos\theta$.

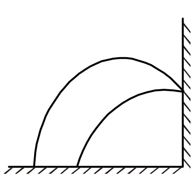
so $V_D = V\cos\theta = V\cos 30^\circ = \frac{\sqrt{3}V}{2}$

Q.37 (C)

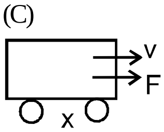
Q.38 (D)
Infinite

Q.39 (B)

$$\frac{2v\cos\theta}{g}$$



Q.40 (C)



$$\vec{v_r} = \vec{v_{mc}}$$

$$v_r = \vec{v_m} - \vec{v_c} = v - u = 0.$$

$\therefore$ since $v_r = 0$ so $F_t = \frac{vrdm}{dt} = 0.$

$$F_{net} = m\frac{dv}{dt}$$

$$F + 0 = (m_0 - \mu t)\frac{dv}{dt}$$

$$\therefore F = (m_0 - \mu t)\frac{dv}{dt}.$$

Q.41 (C)
Neglecting gravity,

$$v = u\ell n\left(\frac{m_0}{m_t}\right);$$

u = ejection velocity w.r.t. balloon.
m_0 = initial mass m_t = mass at any time t.

$$= 2\ell n\left(\frac{m_0}{m_0/2}\right) = 2\ell n 2.$$

Q.42 **((a) & (b))**
(a) Since the speed remains same for both sand and car at same instant
$\therefore$ Momentum is conserved in both A and C point
(b) B
Car maintains the same speed.

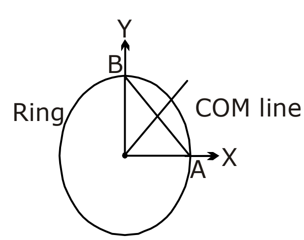
JEE-ADVANCED

MCQ/COMPREHENSION/COLUMN MATCHING

Q.1 (A,B,C,D)

Q.2 (A, B)

Q.3 (B, D)
Center of mass of ring is at centre and centre of mass of chord AB is at its mid point so centre of mass of this combination lie at the line which makes 45° with x axis.



Possible combination

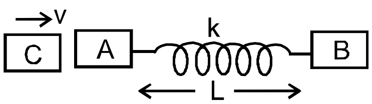
$$\left(\frac{R}{3}, \frac{R}{3}; \frac{R}{4}, \frac{R}{4}\right)$$

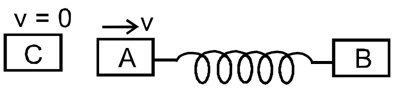
Q.4 (B, C)

Q.5 (B, D)

Q.6 (C, D)

Q.7 (B,D)





$P_i = mv$ (i)
$P_f = (m + m)\, v'$
at maximum compression
$P_i = P_f$ $\quad v' = v/2$
By energy compression

$$\frac{1}{2} mv^2 + 0 = \frac{1}{2}(2m)(v)^2 + \frac{1}{2} kx^2$$

$$kx^2 = \frac{mv^2}{2} \Rightarrow x = \sqrt{\frac{m}{2k}} \times v\,.$$

at maximum compression $k = \frac{1}{2}(m+m)v^2 \Rightarrow k = mv^2$

$= mv^2/4.$

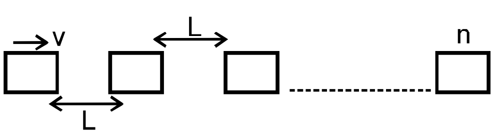
Q.8 (A,D)

Q.9 (B,C)

Q.10 (A,D)

Q.11 (A,B,C,D)

Q.12 (A,C)



$mv = nv'm \Rightarrow v' = \frac{v}{n}$

time for first collisen is $t_1 = \frac{L}{V}$ (2nd block)

2nd collisions $t_2 = \frac{2\ell}{V} = 2t_1$

(3rd block)
so $\quad t = t_1 + 2t_1 + 3t_1 + at_1$ ....$(n–1)\, t_1$.
$t = t_1\, [1 + 2 + 3]$ ....$(n–1)]$

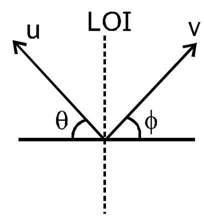
$$= \frac{(n-1)(n-1+1)}{2} = \frac{n(n-1)}{2}$$

so $\quad t = \frac{L}{2x} n\,(n-1).$

Q.13 (A,B,C,D)
$v\cos\phi = u\cos\theta$
$v\sin\phi = eu\sin\theta$

$$v^2 = u\sqrt{\cos^2\theta + e^2\sin^2\theta}$$

$$v = u\sqrt{(1-\sin^2\theta) + e^2\sin^2\theta}$$



$$\therefore\ v = u\sqrt{1-(1-e^2)\sin^2\theta}$$

$\tan\phi = e\tan\theta.$
$I = m\,(v_{LOI} - u_{LOI})$
$= m\,(eu\sin\theta - u\sin\theta)$
$= mu\,(1 + e)\sin\theta.$

$k_i = \frac{1}{2} mu^2 \qquad k_f = \frac{1}{2} mv^2$

$$\frac{k_f}{k_i} = \frac{1/2\, mv^2}{1/2\, mu^2} = \cos^2\theta + e^2\sin^2\theta.$$

Q.14 (A,C,D)

$a = \frac{f}{m}$ for elastic collission e = 1

$v_1^2 = 0 + 2ad$

$v_{b1}^2 = \frac{2F}{m}.d \quad v_{b1} = \sqrt{\frac{2Fd}{m}}$

after collisin $v_{b2} = 0$.

Q.15 (B,D)

Q.16 (B,C)
$u_1 = v\ v_2 = -(v + 2u)$ $\qquad e = 1.$
$|vdt| = m\,(v_1 - u_1)$
$vdt = m\,(+v + 2u + v)$
$vdt = 2m\,(u + v).$

$$v = \frac{2m(u+v)}{dt}.$$

$u_1 = v\ v_1 = 2u + v$

$$\Delta k = \frac{1}{2} mv_1^2 - \frac{1}{2} mu_1^2 = \frac{1}{2}[m\,(2u + v)^2 - v^2]$$

$= \frac{m}{2}\,[4u^2 + v^2 + 4uv - v^2]$

$= 2mu\,(u + v)$

By energy compression

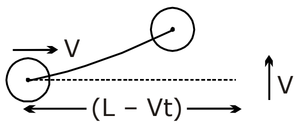
$mv^2 + 0 = (2m) + kx^2$

$kx^2 = \Rightarrow x =.$

at maximum compression

$k = (m+m)v^2 \Rightarrow k = mv^2 = mv^2/4.$

Q.17 (A,C)



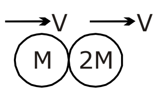
$\sqrt{(Vt)^2 + (L - vt)^2} \le L$

$2V^2t^2 + L^2 - 2LVt \le L^2$

$Vt - L \le 0$

$t \le \frac{L}{V}$

Q.18 (A,B,D)



For minimum kinetic energy

$MV_0 = 3MV \Rightarrow V = V_0/3$

$\therefore \Delta K = -\left[\frac{1}{2}3m\left(\frac{V_0}{3}\right)^2 - \frac{1}{2}mv_0^2\right]$

= 2 Joule

Q.19 (A,B,C)

Momentum conservation

$1 \times 21 - 2 \times 4 = 1 \times 1 + 2 \times V'$

$V' = 6$ m/s

$e = \frac{6-1}{21+4} = \frac{1}{5}$

Loss of kinetic energy $= k_f - k_i$

$= \frac{1}{2} \times 1 \times (1)^2 + \frac{1}{2} \times 2 \times (6)^2$

$- \left(\frac{1}{2} \times 1 \times (21)^2 + \frac{1}{2} \times 2 \times (4)^2\right)$

= 200 J

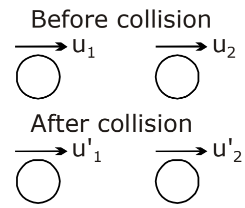
Q.20 (A,B,C,D)

Inelastic collision

$0 < e < 1$

Q.21 (B,D)

Given



$u_2 - u_1 = v_1$ and $u_2' - u_1' = v_2$

$e = \frac{u_2' - u_1'}{u_1 - u_2}$

$\vec{v}_1 = -\vec{v}_2$ (elastic collision, e = 1)

In general for all cases

$\vec{v}_1 = -k\vec{v}_2 \;\; k \ge 1$

Q.22 (C)

(a) The acceleration of the centre of mass is

$$a_{COM} = \frac{F}{2m}$$

The displacement of the centre of mass at time t will be

$$x = \frac{1}{2} a_{COM} t^2 = \frac{Ft^2}{4m}$$ **Ans.**

Q.23 (A)

Q.24 (D)

(Q. 22 and Q. 24)

Suppose the displacement of the first block is x_1 and that of the second is x_2. Then,

$$x = \frac{mx_1 + mx_2}{2m} \quad \text{or,} \quad \frac{Ft^2}{4m} = \frac{x_1 + x_2}{2}$$

or, $x_1 + x_2 = \frac{Ft^2}{2m}$

...(i)

Further, the extension of the spring is $x_1 - x_2$. Therefore,

$x_1 - x_2 = x_0$ ...(ii)

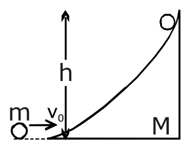
From Eqs. (i) and (ii), $x_1 = \frac{1}{2}\left(\frac{Ft^2}{2m} + x_0\right)$

and $x_2 = \frac{1}{2}\left(\frac{Ft^2}{2m} - x_0\right)$ **Ans.**

Q.25 (B)

As net force in x direction is zero. So from momentum conservation.

$mV_0 = (M+m)V_2$



$$V_2 = \frac{MV_0}{M+m}$$

Q.26 (B,D)

Velocity of center of mass

$$V_{COM} = \frac{MV + mV}{M+m} = V$$

So both are at rest with respect to centre of mass. And kinetic energy is converted into potential energy.

Q .27 (C)

By Energy conservation

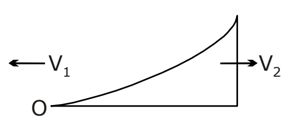
$$\frac{1}{2}mv_0^2 = \frac{1}{2}(M+m)\left(\frac{mv_0}{M+m}\right)^2 + mgh$$

After solving

$$\Rightarrow h = \left(\frac{M}{M+m}\right)\frac{V_0^2}{2g}$$

Q.28 (C)

V_1 is the velocity of particel and V_2 is the velocity of wedge.



$(V_1 + V_2)$ = vel. of particle w.r.t. wedge

$$\Rightarrow \quad -\left(\frac{mV_0 + M(-V_0)}{M+m}\right) + \left(\frac{mV_0 + mV_0}{M+m}\right)$$

$= V_0$

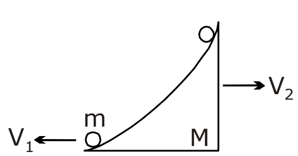
Q.29 (B,C)

As net force in x direction is zero.

So by momentum conservation

$Mv_2 - mv_1 = mV_0$

and $V_1 + V_2 = V_0$



Q.30 (B)

As net force in x direction is zero.

So by momentum conservation

$MV_2 - mV_1 = mV_0$ ....(1)

$V_1 + V_2 = V_0$ ....(2)

By solving

$$V_1 = V_0\left(\frac{M-m}{M+m}\right)$$

Q.31 (A,B,C,D)

(a) $\because V_1 + V_2 = V_0$

$$V_2 = V_0 - V_0\left(\frac{M-m}{M+m}\right)$$

$$= \frac{(M+m)V_0 - V_0M + V_0m}{M+m}$$

$$= \frac{2mV_0}{M+m}$$

$$K.E. = \frac{1}{2} \times M \times \frac{4m^2V_0^2}{(M+m)^2}$$

$$[\because h = \frac{M}{(m+M)}\frac{V_0^2}{2g}]$$

$$\therefore K.E. = \frac{4m^2}{(m+M)}gh$$

(b) $V_2 = \dfrac{2mv_0}{M+m}$

(c) $\Delta K.E. = k_f - k_i$

$$= \frac{1}{2}M\left(\frac{4m^2V_0^2}{(M+m)^2}\right) - 0$$

$$= \frac{4mM}{(m+M)^2}\left(\frac{1}{2}mV_0^2\right)$$

(d) $\because$ vel. of wedge $V_2 = \dfrac{2mV_0}{M+m}$

Vel. of particle $V_1 = V_0\left(\dfrac{M-m}{M+m}\right)$

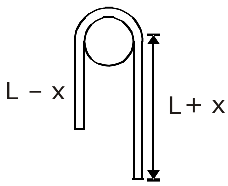
$$V_{COM} = \frac{MV_2 + (-mV_1)}{M+m}$$

$$= \frac{mv_0}{M+m}$$

Q.32 (A)

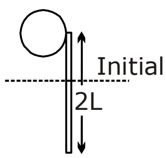
$$a = \frac{m_1 - m_2}{m_1 + m_2} g$$

Let $m_1 = (L + x)\lambda$ and $m_2 = (L - x)\lambda$
where λ is mass per unit length



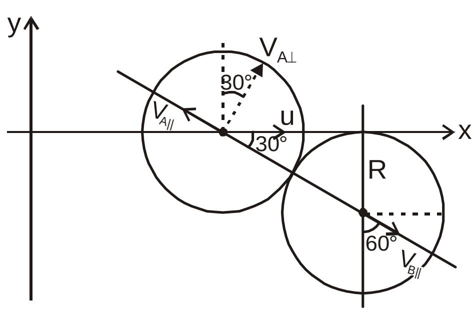
$$a = \frac{2x}{2L} g = \frac{x}{L} g$$

Q.33 (C)
from energy conservation



$$mg\,\frac{\ell}{2} = \frac{1}{2} mv^2 \Rightarrow u = \sqrt{g\ell}$$

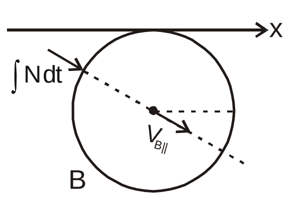
Q.34 (A)
During collision, forces act along line of impact. As collision is elastic and both the balls have same mass, velocities are exchanged along the line of impact. Therefore ball B moves with velocity $V_{B||}$, that is equal to u cos 30°. Ball A moves perpendicular to the line of impact with velocity $V_{A\perp}$ = u cos60°. Along the line of impact, ball A does not have any velocity after the collision.
Therefore velocity of ball A in vector form after the collision



$= V_{A\perp}\cos 60°i + V_{A\perp}\cos 30°j$
$= (u\cos 60°)\cos 60°i + (u\cos 60°)\cos 30°j$

$$= 4.\frac{1}{2}.\frac{1}{2}.i + 4.\frac{1}{2}.\frac{\sqrt{3}}{2}.j = (i + \sqrt{3}j)\ \text{m/s}$$

Q.35 (C)
Using impulse-momentum equation for ball B



$\int N\,dt = \vec{p}_f - \vec{p}_i$ and as $\vec{p}_i = 0$

$\int N\,dt = \vec{p}_f$

$= (mu\cos 30°)\cos 30\,i - (mu\cos 30°)\cos 60°\,j$

$$= m.4.\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2}.i - m.4.\frac{\sqrt{3}}{2}.\frac{1}{2}.j$$

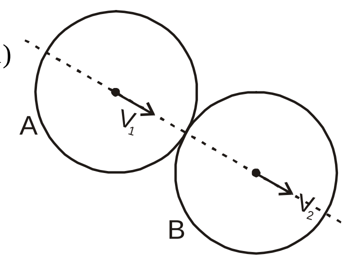
$$= (3mi - \sqrt{3}\,mj)\ \text{kg}\,\frac{m}{s}$$

Q.36 (B)
Suppose V_2 is velocity of ball B along the line of impact and V_1 is velocity of ball A along the line of impact, after the collision, as shown.

Then $\frac{1}{2}$ (Velocity of approach) = Velocity of separation

$$\frac{1}{2}\left[\frac{\sqrt{3}}{2}.u\right] = V_2 - V_1$$

.... (1)



Conserving momentum along the line of impact

$$m.\frac{\sqrt{3}}{2}u = m.V_2 + mV_1$$

.... (2)
Solving and using u = 4 m/s

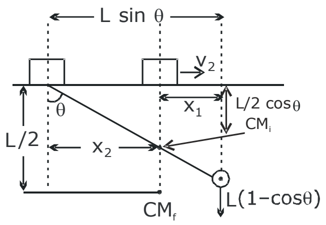
$$V_2 = \frac{3\sqrt{2}}{2}\ \text{m/s}$$

$$\vec{V}_2 = \frac{3\sqrt{3}}{2}\cos 30°i - \frac{3\sqrt{3}}{2}\cos 60°j$$

$$= \left(\frac{9}{4}i - \frac{3\sqrt{3}}{4}j\right)\ \text{m/s}$$

Q.37 (A)
As F_{net} in x direction = 0
$mx_1 = mx_2$ [$\because F_x = 0$]
$x_1 = x_2$
Now $x_1 + x_2 = L\sin\theta$

$$\Rightarrow CM_f = \frac{L\sin\theta}{2}$$



Q.38 (D)

$V_{CMx} = 0$ and $F_x = 0$

from momentum conservation

$mv_1 = mv_2 \Rightarrow v_1 = v_2 = v$(let)

Now energy conservation

$$mg\ell(1 - \cos\theta) = 2\left[\frac{1}{2}mv^2\right]$$

$$v^2 = g\ell(1 - \cos\theta)$$

Distance from centre of mass = $R = \frac{\ell}{2}$

$$\text{So } T = \frac{mv^2}{R} = \frac{mg\ell(1-\cos\theta)}{\ell/2}$$

$T = 2mg(1 - \cos\theta)$

Q.39 (A)

from previous question

$$v_{max} = V = \left[g\ell(1-\cos\theta)\right]^{1/2}$$

Q.40 (B)

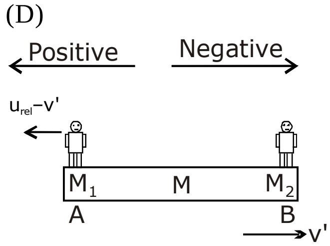
Only in vertical direction

[$\because f_x = 0$ always]

$$\text{So displacement} = \frac{L}{2} - \frac{L}{2}\cos\theta$$

$$= \frac{L}{2}[1 - \cos\theta]$$

Q.41 (D)



By momentum conservation

$O = m_1(u_{rel} - v') - (m_2v' + Mv')$

$m_1(u_{rel} - v') = m_2v' + Mv'$

$$v' = \frac{m_1u_{rel}}{m_1 + m_2 + M}$$

Q.42 (A)

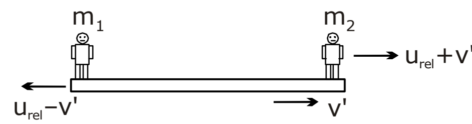
$\vec{F}_{net} = 0$ $\vec{V}_{com} = 0$

$\therefore$ COM is at rest.

$-m_1u + m_2u + Mv = 0$

$$v' = \frac{(m_1 - m_2)}{M}u$$

Q.43 (A)



$m_2(U_{rel} + v') + Mv' = m_1(u_{rel} - v')$

$$v' = \frac{|m_1 - m_2|U_{rel}}{m_1 + m_2 + M}$$

Q.44 (A) p (B) q (C) p,r (D) q,s

(A) If velocity of block A is zero, from conservation of momentum, speed of block B is 2u. Then K.E. of block B = $\frac{1}{2}m(2u)^2 = 2mu^2$ is greater than net mechanical energy of system. Since this is not possible, velocity of A can never be zero.

(B) Since initial velocity of B is zero, it shall be zero for many other instants of time.

(C) Since momentum of system is non-zero, K.E. of system cannot be zero. Also KE of system is minimum at maximum extension of spring.

(D) The potential energy of spring shall be zero whenever it comes to natural length. Also P.E. of spring is maximum at maximum extension of spring.

Q.45 A (q) , (B) p,q (C) r (D) s

(A) Initial velocity of centre of mass of given system is zero and net external force is in vertical direction. Since there is shift of mass downward, the centre of mass has only downward shift.

(B) Obviously there is shift of centre of mass of given system downwards. Also the pulley exerts a force on string which has a horizontal component towards right. Hence centre of mass of system has a rightward shift.

(C) Both block and monkey moves up, hence centre of mass of given system shifts vertically upwards.

(D) Net external force on given system is zero. Hence centre of mass of given system remains at rest.

NUMERICAL VALUE BASED

Q.1 [6 m/s]

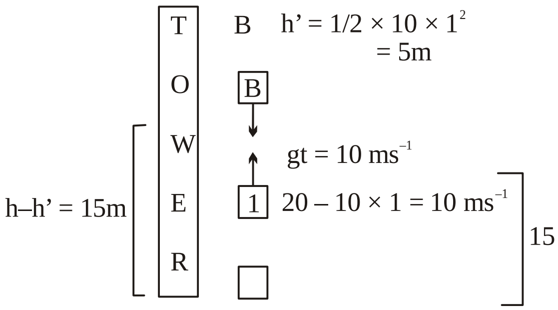
Q.2 [650.00]

Using relative velocity time of slight before collision will be

$$t = \frac{20}{20} = 15$$

By COM at the time of collision

$3 \times 10 - 1 \times 10 = 4 \times v$
$2 \times 10 = 4 \times v$
$5 = v$



$v = 5\ ms^{-1}$

For 1-D motion

$v^2 = u^2 + 2\ as$
$= 5^2 + 2 \times 10 \times 15$
$= 25 + 300 = 325$
$K = 650\ J$

Q.3 [800.00]

By conservation of momentum

$-200 \times v_1 + 2 \times v_2 = 0$
$100\ v_1 = v_2$ ... (1)
$v_1 = v_2 / 100$

$$\left(\frac{v_2}{100}\right) \cdot t = 8$$

$$t = \frac{100}{v_2}$$

$v_2 \cdot t = x$

$$x = v / 2 \cdot \frac{800}{v_2}$$

$x = 800$ m

Q.4 [50]

At the topmost point of the trajectory, the momentum of the system is zero. From conservation of momentum,

$$m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 = 0$$

as $m_1 = m_2 = m_3$

$$\vec{v}_1 + \vec{v}_2 + \vec{v}_3 = 0 \quad .....(1)$$

The second and third fragments reach the ground simultaneously, therefore vertical components of v_2 and v_3 must be same; secondly, v_1 is downwards, the vertical components of v_2 and v_3 are $\frac{-v_1}{2}$ (i.e. directed upwards)

for first fragment, $h = v_1t_1 + \frac{g t_1^2}{2}$ ....(2)

for second fragment, $h = \frac{-v_1t_2}{2} + \frac{g t_2^2}{2}$ ....(3)

from equations (2) & (3), $v_1 = \frac{g(t_2^2 - t_1^2)}{2t_1 + t_2}$

and $h = \frac{g t_1 t_2}{2}\left(\frac{t_1 - 2t_2}{2t_1 + t_2}\right)$

Q.5 [18]

$v = \sqrt{u^2 + 2g(h)}$; $ev = \sqrt{2g(5)}$

$\frac{1}{4} v^2 = 100$; $v^2 = 400$; $v = 20$

$400 = u^2 + 2gh$
$400 = u^2 + 20\ h$ and $h = 3.8$ m
$u^2 = 324 \Rightarrow u = 18$

Q.6 [9]

$$g = \frac{(2 \times 4 + 1)(2)^2}{2(1)(2)(1)^2}$$

$D = 2$
$M = 2$
$t = 1$

$t = 1$ s

$$\left(\frac{m}{2M + m}\right)g = a$$

$v^2 = 0 + 2aH$
$D = vt$
$D^2 = (2aH)\ t^2$

$$\frac{D^2}{2Ht^2} = a = g\left(\frac{m}{m + 2M}\right)$$

$$g = \frac{(m + 2M)D^2}{2Ht^2m}$$

Q.7 [10]

Let displacement of plank be represented by $x\hat{i} + y\hat{j}$

For x-component

$$50[L - x] - 100x - 50x = 0$$

$$x = \frac{L}{4}$$

Similarly y-component

$$50[L - y] - 100y - 50y = 0$$

$$y = \frac{L}{4}$$

Thus $\vec{r} = -\frac{L}{4}\hat{i} - \frac{L}{4}\hat{j}$ or $|\vec{r}| = \frac{L}{4}\sqrt{2}$

Q.8 [50]

By Energy Conservation

$$mg\frac{R}{\sqrt{2}} = \frac{1}{2}\frac{m(\sqrt{2}R)^2\omega^2}{3}$$

$$\Rightarrow \omega^2 = \frac{3g}{\sqrt{2}R}$$

Now, $2N\cos 45° - mg = m \times \frac{3g}{\sqrt{2}R} \times \frac{R}{\sqrt{2}}$

$$\Rightarrow N = \frac{5mg}{2\sqrt{2}} = 50$$

Q.9 [75]

From the principle of conservation of linear momentum we have

$$mu = M_1 v + mv'$$

and $mv' = (m + M_2)v$

or $20u = 1000v + 20v'$

and $20v' = (20 + 2980)v$

or $u = 50v + v'$ ...(i)

and $v' = 150\,v$ ...(ii)

From (i) and (ii), we get $3u = v' + 3v' = 4v'$ or

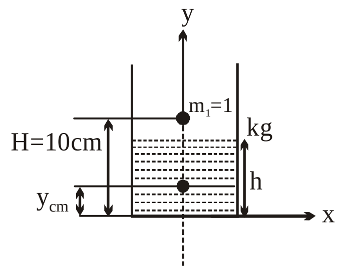
$v' = 3u/4 = 75$

Q.10 [2]

The position of centre of mass of the system is y_{cm}.

$$y_{cm} = \frac{m_1 \times H + m_2 \times \frac{h}{2}}{m_1 + m_2}$$

Where $m_1 = 1$ kg, $m_2 = (0.4 \times h \times 10^3)$ kg



$= (400\,h)$ kg

$$y_{cm} = \frac{1 \times H + (400h) \times \frac{h}{2}}{(1 + 400h)} = \frac{H + 200h^2}{(1 + 400h)}$$

for y_{cm} to be lowest (minimum)

$$\frac{dy_{cm}}{dh} = 0$$

$200h^2 + h - H = 0$

$h = 2$ cm

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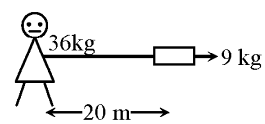
Q.1 (D)



∵ external force does not work on system

So according to concept of centre of mass.

$36x = 9 \times (20 - x)$

Q.2 (B)

$a_B > a_C > a_A$

$a_B = g$

$\vec{a}_{A/CM} = \vec{a}_A - \vec{a}_{CM} (\uparrow)$

$\vec{a}_{B/CM} = \vec{a}_B - \vec{a}_{CM} (\downarrow)$

Q.3 (B)

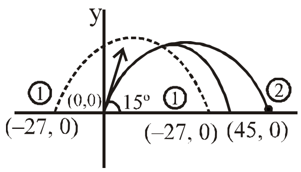
Applying the law of conservation of momentum,

$mv + 0 = (2m)\,v'$

$v' = v/2$

$K.E = \frac{1}{2}(2m)v'^2$

Q.4 (D)



Under influence of constant force centre of mass follows its original path

$$R = \frac{30 \times 30 \times \frac{1}{2}}{10} = 45\text{m}$$

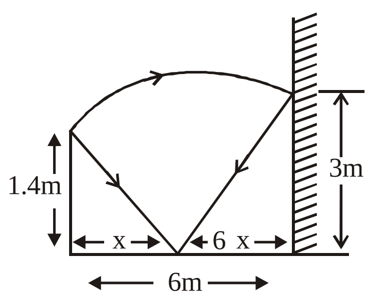
$$45 = \frac{\pm m \times 27 + mx}{m + m}$$

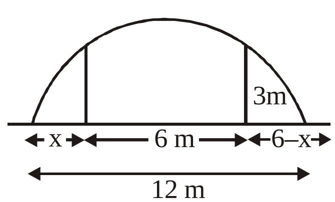
$x = 63, 117\,\text{m}$

Q.5 (D)
Using energy conservation and law of restitution and momentum conservation.

Q.6 (A)
CM will go downwards

Q.7 (A)





$$3 = (6 + x) \tan\theta \left[1 - \frac{6+x}{12}\right]$$

$$3 = \frac{(6+x)(6-x)\tan\theta}{12}$$

$$1.4 = x\tan\theta\left[1 - \frac{x}{12}\right]$$

$$1.4 = \frac{x(12-x)}{12}\tan\theta$$

$$\frac{30}{14} = \frac{36 - x^2}{12x - x^2}$$

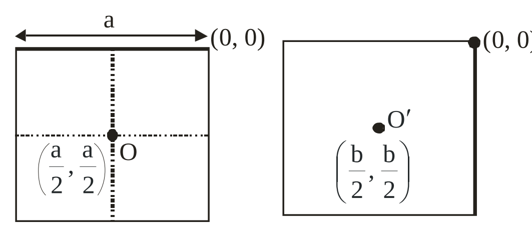
$360\,x - 30\,x^2 = 36 \times 14 - 14x^2$

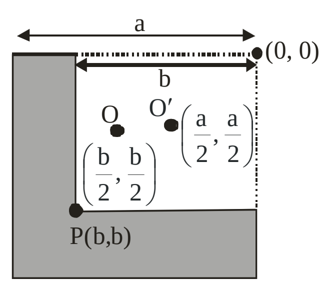
$16x^2 - 360x + 36 \times 14 = 0$

$$x = \frac{360 \pm \sqrt{(360)^2 - 4 \times 36 \times 14 \times 16}}{32}$$

$$x = \frac{360 \pm 312}{32} = \frac{48}{32} = 1.5$$

Q.8 (B)





$$X_{cm} = \frac{m_1x_1 - m_2x_2}{m_1 - m_2}$$

m_1 is the mass of square wooden sheet of side a & m_2 is the mass of removed square portion of side b. x-coordinate of C.O.M. of remaining λ-shaped sheet.
⇒ is areal mass density
⇒ $m_1 = \lambda a^2$, $m_2 = \lambda b^2$

$$X_{cm} = \frac{\lambda(a^2)\left(\frac{a}{2}\right) - \lambda(b^2)\left(\frac{b}{2}\right)}{\lambda a^2 - \lambda b^2}$$

$$X_{cm} = \frac{1}{2}\left(\frac{a^3 - b^3}{a^2 - b^2}\right)$$

similarly $Y_{cm} = \frac{1}{2}\left[\frac{a^3 - b^3}{a^2 - b^2}\right]$

centre of mass lies on point P(b, b)
⇒ X_{cm} = b and Ycm = b
$a^2 + b^2 + ab = 2ab + 2b^2$
$a^2 = ab + b^2$

$$\left(\frac{a}{b}\right)^2 = \left(\frac{a}{b}\right) + 1$$

Let $x = \frac{a}{b}$

$x^2 - x - 1 = 0$

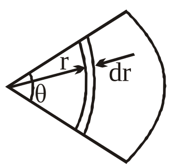
$$x = \frac{1 \pm \sqrt{1+4}}{2}$$

$$x = \frac{\sqrt{5}+1}{2}$$

$$\frac{a}{b} = \frac{\sqrt{5}+1}{2}$$

Q.9 (A)

Planar circular segment can be seen as it consist of Arc element.



Mass of element = dm = $\sigma \times r\theta \times dr$

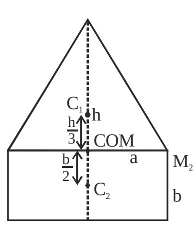
centre of mass of Arc element is at $\frac{r \sin\frac{\theta}{2}}{\frac{\theta}{2}}$

$\therefore$ Centre of mass location of segment

$$= \frac{\Sigma\left(dm\left[\frac{r\sin\theta/2}{\theta/2}\right]\right)}{\Sigma dm} = \frac{\frac{\int_0^R \sigma r\theta dr \times r\sin(\theta/2)}{\theta/2}}{\int_0^R \sigma r\theta dr}$$

$$\Rightarrow 2\left[\frac{\sin\frac{\theta}{2}}{\theta}\right]\frac{R^3}{3\times\frac{R^2}{2}} \Rightarrow \frac{4}{3}R\frac{\sin(\theta/2)}{\theta}$$

Q.10 (C)



Equal are

$\frac{1}{2}ah = ab$

h = 2b

$M_1\frac{h}{3} = M_2\frac{b}{2}$ [centre of mass of combination at the mid-point of their common edge]

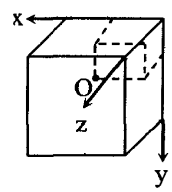
$$\frac{M_1}{M_2} = \frac{3}{2}\frac{b}{h}$$

$$\frac{M_1}{M_2} = \frac{3}{2}\left[\frac{1}{2}\right]$$

$$\frac{M_1}{M_2} = \frac{3}{4}$$

Q.11 (A)

Centre of mass of remaining cube x coordinate = b



$$X_{CM} = \frac{\rho a^3 \times \frac{a}{2} - \rho b^3 \times \frac{b}{2}}{\rho a^3 - \rho b^3}$$

We will consider removed mass as a negative mass

$$b = \frac{\frac{\rho a^4}{2} - \frac{\rho b^4}{2}}{\rho a^3 - \rho b^3}$$

$$a^3 b - b^4 = \frac{a^4}{2} - \frac{b^4}{2}$$

$2a^3b - 2b^4 = a^4 - b^4$

put $a = bx \Rightarrow 2b^4x^3b - 2b^4 = b^4x^4 - b^4$

$2x^3 - 1 = x^4$

$2x^3 - 2 + 1 = x^4$

$2[x^3 - 1] = (x^2-1)(x^2+1)$

$2[x-1][x^2+1+x] = [x-1][x+1][x^2+1]$

$2x^2 + 2 + 2x = x^3 + x + x^2 + 1$

$x^3 - x^2 - x - 1 = 0$

Q.12 **(A)**

Velocity of sand particle just before striking the bottom is v = u + at

v = 0 + 10 × 2 = 20 m/s

pi = $(0.2 \times 10^{-3}) \times 20$

pf = 0

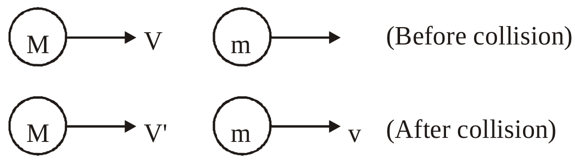
$|\Delta p| = 4 \times 10^{-3}$ k-m/s

$$f_{avg} = \frac{|\Delta p|}{\Delta t} \times n$$

$$= \frac{4 \times 10^{-3} \times 100}{1}$$

$= 0.4\,N$

Q.13 **(D)**



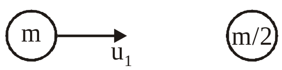
$$e = 1 = \frac{v - V'}{V}$$

$\Rightarrow\; V = v - V'$

$V + V' = v$

Q.14 (B)

Before collision



After collision

By $p_i = p_f$

$$\Rightarrow\; mu, = mv_1 + \frac{M}{2} v_2 \;\;\Rightarrow\; v_1 + \frac{v_2}{2} = u_1 \qquad ....(1)$$

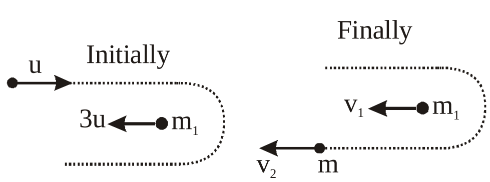
$$\text{By } e = l' = \frac{v_2 - v_1}{u_1} \qquad \Rightarrow\; v_2 - v_1 = u_1 \qquad ......(2)$$

By (1) & (2)

$$v_2 = \frac{4}{3} u_1 \;\&\; v_1 = \frac{u_1}{3}$$

$$\text{So } \frac{v_1}{v_2} = \frac{1}{4}$$

Q.15 **(D)**



from momentum conservation

$-mu + m_1 3u = m_1 v_1 + mv_2$ ....(1)

from energy conservation

$$\frac{1}{2}mu^2 + \frac{1}{2}m_1 9u^2 = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}mv_2^2$$

$$\frac{1}{2}mu^2 + \frac{1}{2}m_1(3u - v_1)(3u + v_1) = \frac{1}{2}mv_2^2$$

from equation ...(1)

$\Rightarrow m_1(3u \,.\, v_1) = m\,(v_2 + u)$

$$\frac{1}{2}mu^2 + \frac{1}{2}m(v_2 + u)(3u + v_1) = \frac{1}{2}mv_2^2$$

as $m_1 >>> m$, we can assume $v_1 \approx 3u$

$u^2 + (v_2 + u)(6u) = v_2^2$

$\Rightarrow v_2 = 7u$

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Q.1 (2)

$$K_1 = \frac{P_1^2}{2m_1} \;\&\; K_2 = \frac{P_2^2}{2m_2}$$

$$\therefore\; \frac{K_1}{K_2} = \left(\frac{P_1}{P_2}\right)^2 \times \left(\frac{M_2}{M_1}\right)$$

$$\therefore\; \left(\frac{P_1}{P_2}\right)^2 = \frac{M_2}{M_1} \qquad \Rightarrow \frac{P_1}{P_2} = \sqrt{\frac{M_2}{M_1}} = \frac{1}{2}$$

Q.2 (3)

Q.3 (2)

$$\frac{M_1}{M_2} = \frac{1}{2}$$

$M_1V_1 = M_2V_2 = P$

$$K_1 = \frac{P^2}{2M_1} \quad K_2 = \frac{P^2}{2M_2}$$

$$\frac{K_1}{K_2} = \frac{M_2}{M_1} = \frac{2}{1}$$

$$= \frac{A}{1} = \frac{2}{1} = 2$$

Q.4 (1)

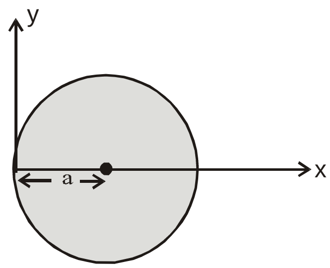
Using linear momentum conservation in y-direction

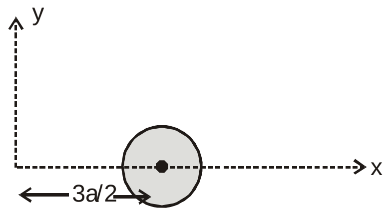
$P_i = 0$

$$P_f = m \times \frac{1}{2} v_1 - m \times \frac{1}{2} v_2$$

$v_1 = v_2$

Q.5 (1)



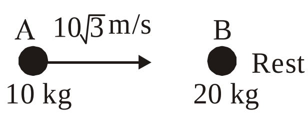


$$x_{com} = \frac{(\sigma \times \pi a^2 \times a) - \sigma\pi\frac{a^2}{4} \times 3\frac{a}{2}}{\sigma\pi a^2 - \sigma\frac{\pi a^2}{4}}$$

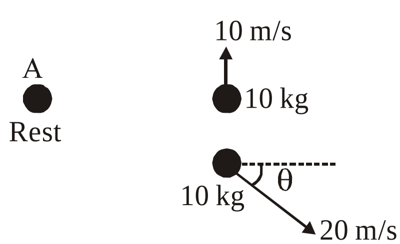
$$x_{com} = \frac{a - \frac{3a}{8}}{1 - \frac{1}{4}} = \frac{\frac{5a}{8}}{\frac{3}{4}} = \frac{5a}{6}$$

Q.6 **(30)**

Before Collision



After Collision



From conservation of momentum along x axis;

$$\vec{P}_i = \vec{P}_f$$

$$10 \times 10\sqrt{3} = 200 \cos\theta$$

$$\cos\theta = \frac{\sqrt{3}}{2}$$

$\theta = 30°$

Q.7 **(3)**

From energy conservation,

$$\left[\frac{\text{after bullet gets embedded till the}}{\text{system comes momentarily at rest}}\right]$$

$$(M + m)g\,h = \frac{1}{2}(M\ m)v_2^1$$

[v_1 is velocity after collision]

$$\therefore\ v_1 = \sqrt{2gh}$$

Applying momentum conservation, (just before and just after collision)

$mv = (M + m)v_1$

$$v = \left(\frac{M+m}{m}\right)v_1 \frac{6}{10 \times 10^{-3}} \times \sqrt{2 \times 9.8 \times 9.8 \times 10^{-2}}$$

≈ 831.55 m / s

Q.8 **(4)**

$v_0 = \sqrt{2gh}$

$v = e\sqrt{gh} = \sqrt{2gh}$

$\Rightarrow e = 0.9$

$S = h + 2e^2h + 2e^4 h + ...$

$$t = \sqrt{\frac{2h}{g}} + 2e\sqrt{\frac{2h}{g}} + 2e^2\sqrt{\frac{2h}{g}} + ...$$

$$v_{av} = \frac{s}{t} = 2.5 \text{ m/s}$$

Q.9 (1)

C comes to rest

V_{cm} of A & B $= \frac{v}{2}$

$$\Rightarrow \frac{1}{2} \text{is } v_{ret}^2 = \frac{1}{2}kx^2$$

$$x = \sqrt{\frac{\mu \times v^2}{k}} = \sqrt{\frac{m}{2k}}v$$

Q.10 (4)

C.O.M of quarter disc is at $\frac{4a}{3\pi}, \frac{4a}{3\pi}$

= 4

Q.11 (20)

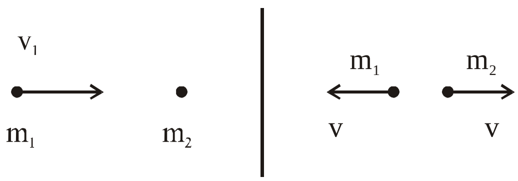
Let velocity of 2[nd] fragment is $\vec{v}$ then by conservation of linear momentum

$$10\left(10\sqrt{3}\right)\hat{i} = (10)\left(10\,\hat{j}\right) + 10\vec{v}$$

$$\Rightarrow \vec{v} = 10\sqrt{3}\hat{i} - 10\hat{j}$$

$$|\vec{v}| = \sqrt{300+100} = \sqrt{400} = 20\,m/s$$

Q.12 (1)



$$m_1v_1 = -m_1v + m_2v$$

$$v_1 = -v + \frac{m_2}{m_1}v$$

$$\frac{(v_1+v)}{v} = \frac{m_2}{m_1}$$

$$e = \frac{2v}{v_1} = 1$$

$$v = \frac{v_1}{2}$$

$$\frac{v_1 + v_1/2}{v_1/2} = \frac{m_2}{m_1}$$

$$3 = \frac{m_2}{m_1}$$

Q.13 (4)

Q.14 (1)

Q.15 (2)

Q.16 (25)

Q.17 (25)

Q.18 (3)

Q.19 [2]

Q.20 (3)

Q.21 (3)

Q.22 [1]

Q.23 (50)

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Q.1 **(D)**

$$R = u\sqrt{\frac{2h}{g}}$$

$$\Rightarrow 20 = V_1\sqrt{\frac{2\times5}{10}} \text{ and } 100 = V_2\sqrt{\frac{2\times5}{10}}$$

$\Rightarrow V_1 = 20$ m/s, $V_2 = 100$ m/sec.

Applying momentum conservation just before and just after the collision

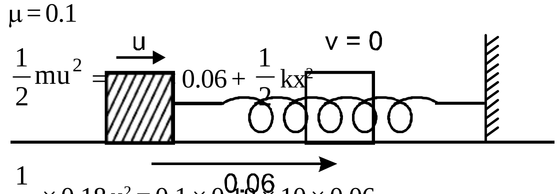
$(0.01)(V) = (0.2)(20) + (0.01)(100)$

$V = 500$ m/s

Q.2 **(4)**

$\mu = 0.1$

$$\frac{1}{2}mu^2 = 0.06 + \frac{1}{2}kx^2$$

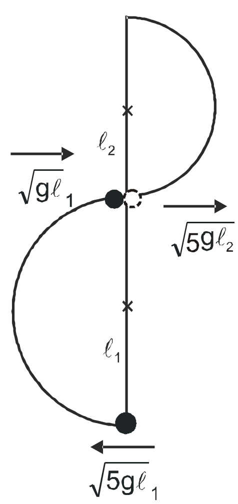


$$\frac{1}{2}\times 0.18\,u^2 = 0.1\times0.18\times10\times0.06$$

$$0.4 = \frac{N}{10}$$

N = 4 **Ans.**

Q.3 **(5)**



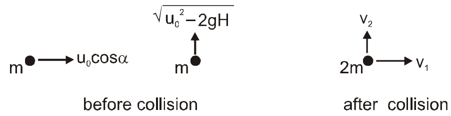
To complete the vertical circle

$$\sqrt{g\ell_1} = \sqrt{5g\ell_2}$$

$$\frac{\ell_1}{\ell_2} = 5$$

Q.4 **(A)**

At the highest point



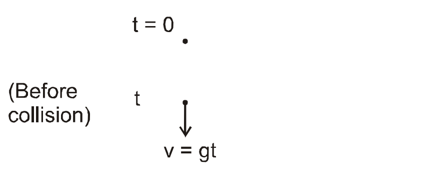
$v_1 = \dfrac{u_0\cos\alpha}{2}$ (by applying momentum conservation in horizontal direction)

$v_2 = \dfrac{u_0\cos\alpha}{2}$ (by applying momentum conservation in vertical direction)

$\left(H = \dfrac{u_0^2\sin^2\alpha}{2g}\right)$

$\theta = 45°$

Q.5 **(B)**



$K = \dfrac{1}{2}mg^2t^2$

$K \propto t^2$: parabolic graph

then during collision kinetic energy first decreases to elastic potential energy and then increases.
Most appropriate graph is B.

Q.6 **(AB)**

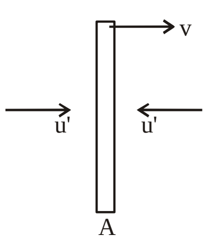
If speed of point mass is v, then using conservation of linear momentum $V = \dfrac{mv}{M}$

$$mgR = \frac{1}{2}mv^2 + \frac{1}{2}M\left(\frac{mv}{M}\right)^2 ; mgR = \frac{1}{2}mv^2\left(1+\frac{m}{M}\right)$$

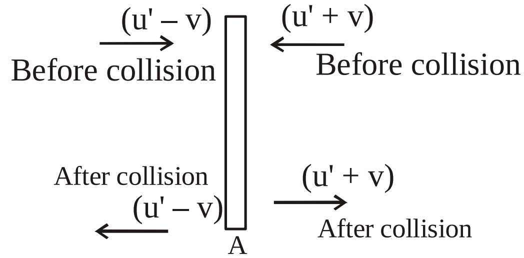
$$v = \sqrt{\frac{2gR}{1+\frac{m}{M}}} \quad ; \; X_M = -\left(\frac{mR}{M+m}\right)$$

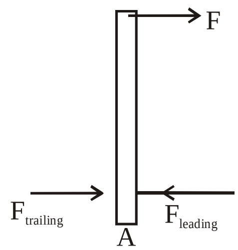
Q.7 **(ABC)**

$u' = \alpha u$, α = constant



w. r. t plane





$F_{trailing} = 2\rho A(u' - v)^2$

$F_{leading} - F_{trailling} = 2\rho A(4u'v) = 8\rho Au'v$

Pressure difference

$$= \frac{F_{leading} - F_{trailing}}{Area} = 8\rho u'v = 8\rho\alpha uv$$

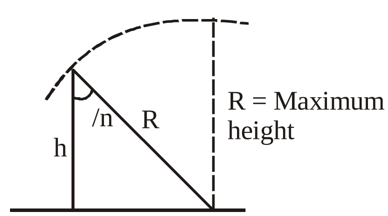
Net force on plate

$$F_{net} = F - 8\rho A\alpha uv = \frac{mdv}{dt}$$

After long time v will be sufficient so $F = 8\rho A\alpha uv$

After that v = constant, i.e. plate will achieve terminal velocity.

Q.8 **(D)**

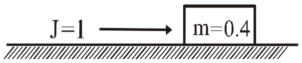


$$\cos\left(\frac{\pi}{n}\right) = \frac{h}{R}$$

$$\Delta = R - h = \frac{h}{\cos\left(\frac{\pi}{n}\right)} - h$$

$$= h\left[\frac{1}{\cos\left(\frac{\pi}{n}\right)} - 1\right]$$

Q.9 [6.30]



$v = v_0 e^{-t/\tau}$

$v_0 = \frac{J}{m} = 2.5\,m/s$

$v = v_0 e^{-t/\tau}$

$\frac{dx}{dt} = v_0 e^{-t/\tau}$

$\int_0^x dx = v_0 \int_0^\tau e^{-t/\tau} dt \qquad \int e^{-x} dx = \frac{e^{-x}}{-1}$

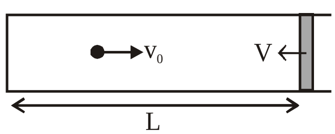
$x = v_0 \left[\frac{e^{-t/\tau}}{-\frac{1}{\tau}} \right]_0^\tau$

$x = 2.5(-4)\left(e^{-1} - e^0\right)$

$x = 25(-4)(0.37 - 1)$

$x = 6.30$

Q.10 (B,C)



(1) average rate of collision = $\frac{2L}{v}$

(2) speed of particle after collision = $2V + v_0$

change in speed = $(2V + v_0) - v_0$

After each collision = 2V

no. of collision per unit time (frequency) = $\frac{v}{2L}$

change in speed in dt time = 2V × number of collision in dt time

$\Rightarrow dv = 2V \left(\frac{v}{2L}\right) . \frac{dL}{V}$

$dv = \frac{vdL}{L}$

Now, $dv = -\frac{vdL}{L}$ {as L decrease}

$\int_{v_0}^{v} \frac{dv}{v} = -\int_{L_0}^{L_0/2} \frac{dL}{L}$

$\Rightarrow [\ln v]_{v_0}^{v} = -[\ln L]_{L}^{L_0/2}$

$\Rightarrow v = 2v_0$

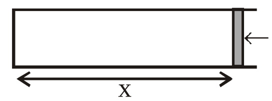
$\Rightarrow KE_{L_0} = \frac{1}{2} mv_0^2$

$\boxed{\frac{KE_{L_0/2}}{KE_0} = 4}$

$KE_{L_0/2} = \frac{1}{2} m(2v_0)^2$

or

$(dt)\left(\frac{v}{2x}\right)\frac{2mv}{dt} = F$



$F = \frac{mv^2}{x}$

$-m\,v\frac{dv}{v} = \frac{mv^2}{x}$

$-\frac{dv}{v} = \frac{dx}{x}$

$ln \frac{v_2}{v_1} = \ln \frac{x_1}{x_2}$

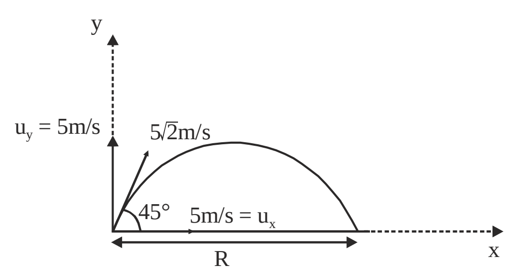
vx = constant ⇒ on decreasing lenth to half K.E. becomes 1/4

vdx + xdv = 0

Question Stem for Question Nos. 11 and 12

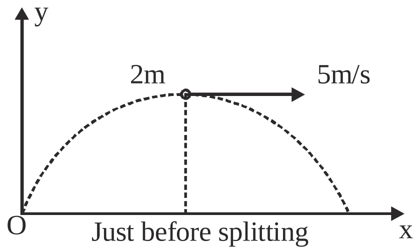
Q.11 [0.50]

Q.12 [7.50]

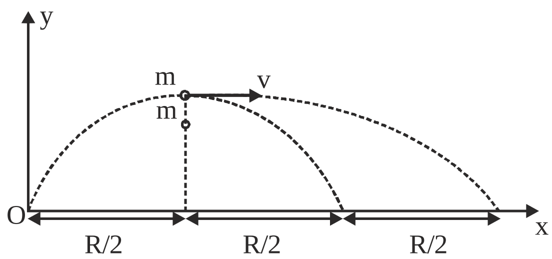


Range $R = \frac{2u_x u_y}{g} = \frac{2\times5\times5}{10} = 5m$

Time of flight $T = \frac{2u_y}{g} = \frac{2\times5}{10} = 1sec$



Just before splitting



Just before splitting

$\therefore$ Time of motion of one part falling vertically downwards is = 0.5 sec = $\frac{T}{2}$

$\Rightarrow$ Time of motion of another part, t = $\frac{T}{2}$ = 0.5 sec

From momentum conservation $\Rightarrow P_i = P_f$

$2m \times 5 = m \times v$

$v = 10$ m/s

Displacement of other part in 0.5 sec in horizontal direction = $v\frac{T}{2}$

$= 10 \times 0.5 = 5$ m = R

$\therefore$ Total distance of second part from point 'O' is,

$x = \frac{3R}{2} = 3 \times \frac{5}{2}$

x = 7.5 m

$\Rightarrow$ t = 0.5 sec

Rotational Motion

EXERCISES

ELEMENTARY

Q.1 (2)

$\theta = \omega t$

$$\theta = \frac{27 \times 3000}{60} \times 1$$

Q.2 (3)

$$\omega = \frac{2\pi}{60} = \frac{\pi}{30} \text{ rad/s}$$

Q.3 (1)

$T = \frac{2\pi}{\omega}$ if time period same then ω will be same

$a_C = \omega^2 r \quad \frac{a_1}{a_2} = \frac{r_1}{r_2}$

Q.4 (4)

$f = 300\frac{\text{rot}}{\text{min}}$; ratation in 1 sec = $\frac{300}{60}$ = 5

angle described in sec = 5 × 2π = 10 π.

Q.5 (4)

Relation between linear acceleration (a_t) and angular acceleration (α) is :-

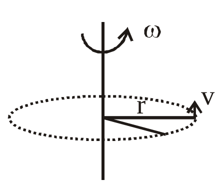
$a_t = \alpha \times R$

so, $R = \frac{a_t}{\alpha} = \frac{10}{5}$ = 2m

Q.6 (1)

$v = \omega r$

r is perpendicular distance of particle from rotational axis so correct option (1).



$v = \omega r$ r is perpendicular distance of particle from rotational axis so correct option (1).

Q.7 (4)

$\vec{v} = \vec{\omega} \times \vec{r}$

from above we get $\omega = \frac{v}{r}$ but $\omega = \frac{d\theta}{dt}$ is not depend on distance (r) from axis of rotation.

Q.8 (3)

As disc is lying in the x-z plane, so applying perpendicular axis theorem :-

$I_x + I_z = I_y$

$30 + I_z = 40$

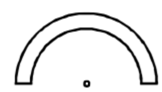
$\Rightarrow I_z = 40 - 30 = 10 \text{ kg m}^2$

Q.9 (2)

mr^2

Q.10 (4)

Q.11 (1)



$$I = \frac{2Mr^2}{2} = Mr^2$$

Q.12 (3)

Given, $I_{\text{solid sphere}} = I_{\text{hallow sphere}}$

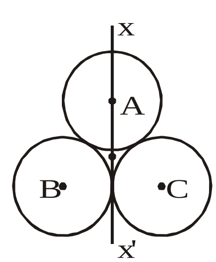
$$\Rightarrow \frac{2}{5}Mr_1^2 = \frac{2}{3}Mr_2^2$$

$$\Rightarrow \frac{r_1^2}{r_2^2} = \frac{5}{3}$$

$$\Rightarrow \frac{r_1}{r_2} = \sqrt{5} : \sqrt{3}$$

Q.13 (3)

$I = I_A + I_B + I_C$



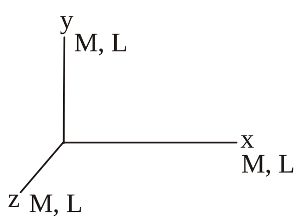
$$I_A = \frac{2}{5}MR^2$$

$$I_B = I_C = \frac{2}{5}MR^2 + MR^2 = \frac{7}{5}MR^2$$

$$\Rightarrow \quad I = \frac{2}{5}MR^2 + \frac{7}{5}MR^2 + \frac{7}{3}MR^2$$

$$I = \frac{16}{5}MR^2$$

Q.14 (2)



$I = I_1 + I_2 + I_3$

$$= \frac{ML^2}{3} + \frac{ML^2}{3} + 0 = \frac{2}{3}ML^2$$

Q.15 (1)

given, $\omega_0 = 20$ rad/sec

$\omega = 0$

$I = 50$ kg-m^2

$t = 10$ sec

$$\propto = \frac{\omega - \omega_0}{t} = \frac{0 - 20}{10} = -2\text{rad/sec}^2$$

and $\tau = I \propto = 50 \times 2 = 100$ kg-m^2/s^2

$= 100$ N-m

Q.16 (4)

Given ; [τ = 2N.m., α = 2 rad/sec^2, k = 2m]

$$I = \frac{2}{2} = 1\text{kg.m}^2$$

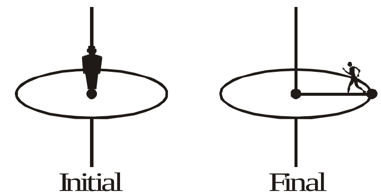
$I = MK^2$

$I = m.2^2$; $m = \frac{1}{4}$.Kg

Q.17 (3)

$$z = 0, \frac{dL}{dt}, L_i = L_f$$

Q.18 (1)



Applying conservation of angular momentum :-

$I_1\omega_1 = I_2\omega_2$

$$\Rightarrow \quad I_1 \times \frac{2\pi}{T_1} = I_2\omega_2$$

$$\Rightarrow \quad 100 \times \frac{2\pi}{10} = [100 + 50 \times (2)^2] \times \omega_2$$

on solving, $\omega_2 = \frac{2\pi}{30}$ rad/sec

Q.19 (3)

From L = constant

L = Iω

Because $\tau_{ext} = 0$

Due to drop the wax on disc moment of inertia of its will be increase so will be decrease.

Q.20 **(2)**

$\tau_{ext} = 0$, So L = Iω = constant when girl moves from edge towards centre I will decrease, and 'ω' will increase.

Q.21 (2)

$$f = \frac{\frac{1}{2}I\omega^2}{\frac{1}{2}I\omega^2 + \frac{1}{2}mv^2}$$

where $v = \omega r$ and $I = I = \frac{2}{5}mR^2$

Q.22 (3)

Because sphere has maximum translational lurchi energy first decrease in Potential energy.

Q.23 (2)

Acceleration of a purely rolling object on an inclined plane is :-

$$a = \frac{g\sin\theta}{\left(1+\frac{K^2}{R^2}\right)}$$

for spherical shell, $\frac{K^2}{R^2} = \frac{2}{3}$

for solid cylinder, $\frac{K^2}{R^2} = \frac{1}{2}$

so, $\frac{a_{shell}}{a_{cylinder}} = \frac{\frac{g\sin\theta}{\left(1+\frac{2}{3}\right)}}{\frac{g\sin\theta}{\left(1+\frac{1}{2}\right)}} = \frac{9}{10}$

Q.24 (1)

$$\text{Fraction} = \frac{K_{Rotation}}{K_{Total}} = \frac{\frac{K^2}{R^2}}{1+\frac{K^2}{R^2}}$$

For disc, $\frac{K^2}{R^2} = \frac{1}{2}$,

so, fraction $= \frac{\frac{1}{2}}{1+\frac{1}{2}} = 1:3$

JEE-MAIN
OBJECTIVE QUESTIONS

Q.1 (2)

ω_0 = 3000 rad/min

$\omega_0 = \frac{3000}{60}$ rad/sec = (50 rad/sec)

t = 10 sec

$\omega_f = 0$

$\omega_f = \omega_0 + \alpha t$

$\theta = 50 - \alpha\,(10)$

α = 5 rad/sec^2

$\theta = \omega_o t + \frac{1}{2}\alpha t^2$

$\theta = (50)(10) + \frac{1}{2}(-10)(10)^2$

θ = 500 – 250 = 250 rad

Q.2 (3)

V = ωR

V = 10 × 0.2 = 2m /sec.

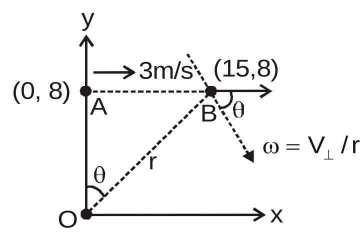
Q.3 **(3)**

$\omega = V_\perp/r$

$\omega = 3\cos\theta/r$

$\omega = \frac{3}{r}\times\frac{8}{r} = \frac{24}{r^2}$

In ΔOAB



$r^2 = (15)^2 + (8)^2 = 289$

$\omega = \frac{24}{289}$ rad/s

Q.4 (3)

$m_A = (\sigma.\pi r^2.t)$

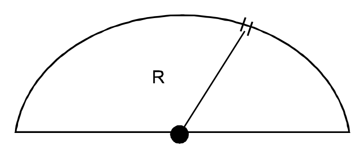
$m_B = \sigma.\pi\,(2r)^2\,(t/2) = (\sigma 2\pi\rho^2 t)$

$m_B > m_A$

$R_B > R_A$

so, $I_B > I_A$

Q.5 (1)



$I = \int dmr^2$

$I = r^2\int dm = r^2 m = mr^2$

Q.6 (1)

$\sigma_B > \sigma_A$

$I_B > I_A$

so, If the axes are parallel $I_A < I_B$

Q.7 (3)

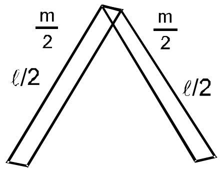
$I_2 = I_1 + Md^2$ Then $I_2 > I_1$

Q.8 **(4)**

Moment of inertia of the elliptical disc should be less than that of a circular disc having radius equal to the major axis of the elliptical disc.

Hence (4)

Q.9 (3)



$I_0 = I_1 + I_2$

$$I_0 = \frac{(m/2)\left(\frac{\ell}{2}\right)^2}{3} + \frac{(m/2)\left(\frac{\ell}{2}\right)^2}{3} = \frac{m\ell^2}{12}$$

Q.10 (3)

$I_x + I_y = I_z$

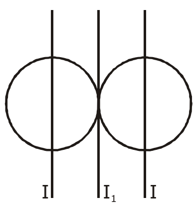
$2I_x = I_z$

$\therefore \quad I_I = 2 \times 200 = 400 \text{ gm cm}^2$.

Q.11 **(4)**

Moment of inertia of a body depends upon mass and distribution of mass about the axis.

Q.12 (2)



Moment of inertia about

diameter of sphere $I = \frac{2}{5}mr^2$

Moment of inertia about tangent at their common

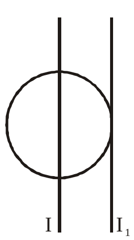
point $I_1 = \left(\frac{2}{5}mr^2 + mr^2\right) \times 2 = \frac{14}{5}mr^2$ $I_1 = 7I$

Q.13 (4)

Moment of inertia of disc

about diameter $I = \frac{mr^2}{4} = 2,$

$mr^2 = 8$



Moment of inertia about the axis through a point on rim.

$$I_1 = \frac{mr^2}{4} + mr^2 = 10$$

Q.14 (1)

Moment of inertia of solid sphere $I_1 = \frac{2}{5}mr_1^2$

Moment of inertia of hollowsphere $I_2 = \frac{2}{3}mr_2^2$

$$\frac{2}{m}mr_1^2 = \frac{2}{3}mr_2^2 \Rightarrow \frac{r_1}{r_2} = \sqrt{\frac{5}{3}}$$

Q.15 (1)

M inertia about yy' axis are

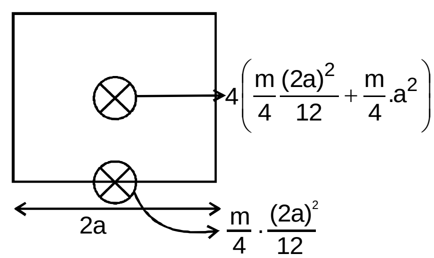
$I = I_1 + I_2 + I_3 = 2I_1 + I_3$ ($\because I_1 = I_2$)

$$\frac{MR^2}{2} + 2\left(\frac{MR^2}{2} + MR^2\right)$$

$\left[I = \left(\frac{MR^2}{2} + MR^2\right)\right]$ applying parallel axis theorem = 7/2 MR^2 = 7/2 PQ^2.

Q.16 **(4)**

(1) $\frac{Ma^2}{2}$ (disc)



(2) $Ma^{2(\text{Ring})}$

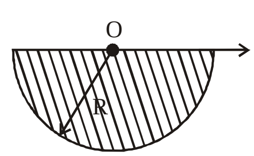
(3) $\frac{2}{3}Ma^{2(\text{square lamina})}$

(4) 4 Rods forming a square of side 2a.

$$\therefore 4\left(\frac{m}{4}\frac{(2a)^2}{12} + \frac{m}{4}.a^2\right)$$

$$= \frac{ma^2}{3} + ma^2 = \frac{4ma^2}{3}$$

Q.17 (2)



$I = \frac{MR^2}{2}$

(pasing through 0)

Q.18 (4)
M.O.I. about C.O.M. is Minimum
$I = I_{C.M.} + Mx_0^2$
$I = 2x^2 - 12x + 27$

$$\therefore \frac{dI}{dx} = 4x - 12 = 0$$

$\Rightarrow x = 3$

Q.19 (2)

$$\tau = I\alpha = \frac{mr^2}{2} \times \alpha$$

$\alpha = 0.25 \text{rad/sec}^2$

Q.20 (2)
$\tau = I\alpha$
τ = constant $\Rightarrow \omega$ = increases

Q.21 (4)
$\omega = \omega_0 + \alpha t$
$100 = 10 + \alpha(15) \Rightarrow \alpha = 6$ rad/sec^2
$\tau = I\alpha \Rightarrow 60$ Nm

Q.22 (4)
$\tau = I\alpha$
$2 = I \times 2 \Rightarrow I = 1$ kgm^2
$I = MR^2$
$1 = M(2)^2$

$$M = \frac{1}{4} \text{kg}$$

Q.23 (1)

$$\tau = I\alpha = (mr^2)\alpha$$

Now, $\tau_1 = (2m)\frac{r^2}{4} \times \alpha = \frac{mr^2}{2} \times \alpha \Rightarrow \tau_1 = \frac{\tau}{2}$

Q.24 (1)
$\tau = I\,\alpha$
$\tau_A = \tau_B$
$I_A\,\alpha_A = I_B\,\alpha_B$
$I_A < I_B$
$\alpha_A > \alpha_B$
$\omega_A > \omega_B$

Q.25 **(1)**

$\vec{F}_1 = 2i + 3j + 4k$

$\vec{F}_2 = -2i - 3j - 4k$

Net force $\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 = 0$ the body is in translational equilibrium.

$\vec{r}_1 = 3i + 3J + 4k$ $\quad\quad$ $\vec{r}_2 = i$

$\vec{\tau}_2 = \vec{r}_1 \times \vec{F}_1$

$= (3\hat{i} + 3\hat{j} + 4\hat{k}) \times (2\hat{i} + 3\hat{j} + 4\hat{k})$

$\vec{\tau}_1 = 9\hat{k} - 12\hat{j} - 6\hat{j} + 12\hat{i} + 8\hat{j} - 12\hat{i}$

$\vec{\tau}_1 = -4\hat{j} + 3\hat{k}$

sw $\quad \vec{\tau}_2 = \vec{r}_2 \times \vec{F}_2 = (\hat{i}) \times (-2\hat{i} - 3\hat{j} - 4\hat{k})$

$= -3\hat{k} + 4\hat{j}$

$\left(\vec{\tau}_1 + \vec{\tau}_2 = -4\hat{i} + 3\hat{k} - 3\hat{k} + 4\hat{j} = 0\right)$

body in rotational equilibrium

Q.26 (3)

$F = 4\hat{i} - 10\hat{j}$

$\vec{r} = (-5\hat{i} - 3\hat{j})$

$\tau = \vec{r} \times \vec{F}$

$= (-5\hat{i} - 3\hat{j}) \times (4\hat{i} - 10\hat{j})$

$= 50\hat{k} + 12\hat{k} = 62\hat{k}$

Q.27 **(3)**

$\vec{F} = 2\hat{i} + 3\hat{j} - \hat{k}$ at point $(2,-3,1)$

torque about point $(0, 0, 2)$

$\vec{r} = (2\hat{i} - 3\hat{j} + \hat{k}) - 2\hat{k}$

$\vec{\tau} = \vec{r} \times \vec{F} = (2\hat{i} - 3\hat{j} - \hat{k}) \times (2\hat{i} + 3\hat{j} - \hat{k})$

$\vec{\tau} = (6\hat{i} + 12\hat{k})$

$|\vec{\tau}| = (6\sqrt{5})$

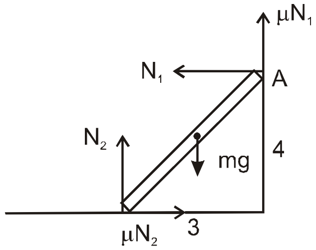
Q.28 (3)
torque of a couple is always remains constant about any point

Q.29 (2)
Torque about O
F × 40 + F × 80 – (F × 20 + F × 60)
In clockwise direction
= F × 40

Q.30 **(3)**
$N_1 = \mu N_2$,

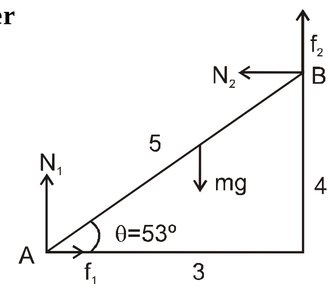
$\mu N_1 + N_2 = mg$, $\tau_A = o \Rightarrow$

$$3 N_2 - 4 N_1 - \frac{3}{2} mg = o$$



Hence $\mu = \frac{1}{3}$ **Ans.**

Aliter



Using force balance

$f_1 = -\mu N_1$ ——(1) $\qquad N_1 + f_2 = mg$

$f_2 = \mu N_2$ $\qquad N_2 = f_1$

$N_2 = \mu N_1$ ——(2)

Using aq (1)

$N_1 + \mu N_2 = mg$

$N_1 + \mu N_1 = mg$

$$N_1 + \left(\frac{mg}{1+\mu^2}\right)$$

torque about point B $\Rightarrow \tau_B = 0$ For rotational equilibrium

$f_1 \times 4 + mg\,(5/2 \cos 53^o) = 3N_1$

$$4\mu N_1 + \frac{3mg}{2} = 3N_1$$

$$\frac{3mg}{2} = (3 - 4\mu)\, N_1$$

$$\frac{3mg}{2} = (3 - 4\mu)\left(\frac{mg}{1+\mu^2}\right)$$

$$\frac{3}{2} = \left(\frac{3-4\mu}{1+\mu^2}\right)$$

$3 + 3\mu^2 = 6 - 8\mu$

$3\mu^2 + 8\mu - 3 = 0$

$3\mu^2 + 9\mu - \mu - 3 = 0$

$3\mu(\mu + 3) - 1\,(\mu + 3)$

$\Rightarrow (\mu = 1/3)$

Q.31 (2)

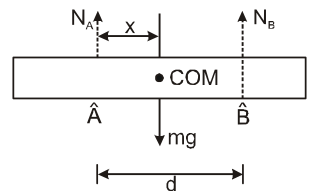
As shown in FBD $\rightarrow$ Equation in verticle direction

$N_A + N_B = mg$

Taking moments about 'A'

$mg.x = d.N_B$

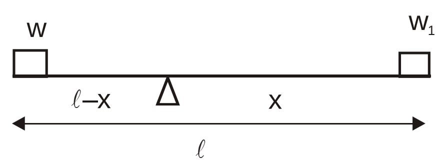
$$N_B = \frac{mg.x}{d}$$



$N_A = mg - N_B$

$$N_A = mg - \frac{mg\,x}{d} = mg\left(\frac{d-x}{d}\right) .= w\left(1-\frac{x}{d}\right)$$

Q.32 **(1)**



weight of object = w

$w\,(\ell - x) = w_1 x$ ....(i)

If weight is kept in another pan then :

$w_2(\ell - x) = wx$ ....(ii)

By (i) & (ii)

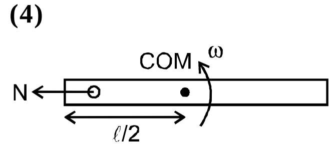
$$\frac{w}{w_2} = \frac{w_1}{w} \Rightarrow w^2 = w_1 w_2$$

$$w = \sqrt{w_1 w_2}\ .$$

Q.33 (3)

Body is rotating uniformly so resultant force on particale is centripetal force which is horizontal and intercecting the axis of rotation.

Q.34 **(4)**



$$N = \left(m\omega^2 \frac{\ell}{2}\right)$$

Q.35 (1)

Initial velocity of each point onthe rod is zero so angular velocity of rod is zero.

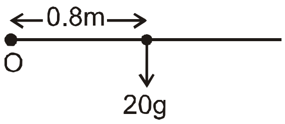
Torque about O

$\tau = I\,\alpha$

$$20g\,(0.8) = \frac{m\ell^2}{3}\alpha \Rightarrow \quad 20g\,(0.8) = \frac{20(1.6)^2}{3}\alpha$$

$$\Rightarrow \frac{3g}{3.2} = \alpha = \text{angular acceleration}$$

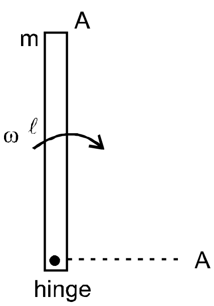
$$\Rightarrow \quad \alpha = \frac{15\,g}{16}$$



Q.36 (3)

Beam is not at rotational equilibrium, so force exerted by the rod (beam) decrcase

Q.37 **(2)**



using energy conservation

$$mg\,\frac{\ell}{2} = \frac{1}{2}\,I\omega^2$$

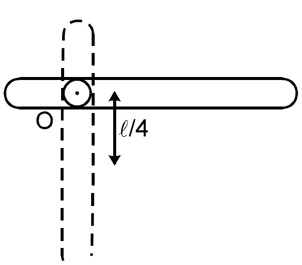
$$mg\,\frac{\ell}{2} = \frac{1}{2}\cdot\frac{m\ell}{3}\,\omega^2$$

$$\ell = 1m\ \omega = \sqrt{\frac{3g}{\ell}}$$

$$V_A = \omega\ell = \sqrt{3g} = \left(\sqrt{3g}\right)$$

Q.38 **(3)**

By energy conservation



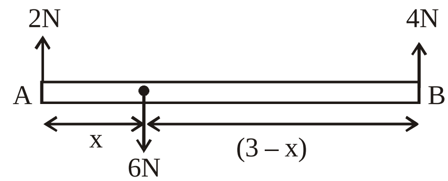
$$mg\,\frac{\ell}{4} = \frac{1}{2}\cdot\left(\frac{7}{48}m\ell^2\right)\omega^2$$

$$[\ I_{(about\ O)} = \frac{m\ell^2}{12} + m\left(\frac{\ell}{4}\right)^2$$

$$I_0 = \frac{7}{48}\,ml^2 \Rightarrow \omega = \sqrt{\frac{24g}{7\ell}}\ \textbf{Ans.}$$

Q.39 (4)

Torque about B



$2 \times 3 = 6\,(3 - x)$

$6 = 18 - 6x$

$6x = 12$

$x = 2m$

Q.40 **(3)**

By torque balance

$16\,L_1 = mL_2$ ....(1)

$mL_1 = 4L_2$ ....(2)

$16 \times 4 = m^2$

$m = 8kg$

Q.41 **(1)**

$\tau = I\alpha$

α = constant

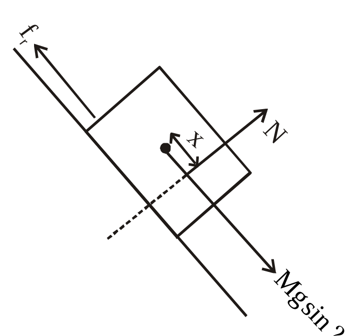
Its angular velocity increases

But force on hinge is constant

Q.42 **(3)**

$$\tau_{avg} = \frac{\Delta L}{\Delta T} = \frac{10}{2} = 5N\text{-}m$$

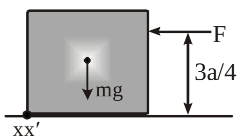
Q.43 (4)



$f_r = Mg \sin\theta = \mu Mg \cos\theta \; f_r \cdot \frac{a}{2} = N.x = \tau_N$

$$\boxed{Mg\frac{a}{2}\sin\theta = \tau_N}$$

Q.44 (1)



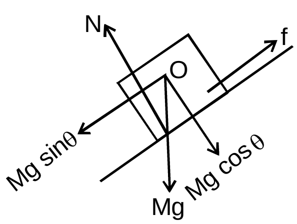
For topling about edge xx′ __________

$F_{min.}\frac{3a}{4} = mg\frac{a}{2}$

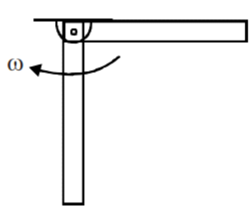
$F_{min.} = \frac{2mg}{3}$.

Q.45 **(1)**

To Balance torque N shifts Downwards



Q.46 (2)



By work energy theorem

$$my\frac{L}{2} = \frac{1}{2}\frac{mL^2}{3}\omega^2$$

$$\omega = \sqrt{\frac{3g}{L}}$$

Q.47 (4)

$\frac{1}{2}I\omega^2 = 1000$

$\omega = 10\,\text{rad/sec}$

$2\pi f = 10 \Rightarrow f = \frac{5}{\pi}\text{rad/sec} = \frac{300}{\pi}\text{rad/min}$

Q.48 **(2)**

$P = \vec{\tau} \cdot \vec{\omega} \Rightarrow P = \tau\,\omega$

P= τ α t

Constant

Q.49 (1)

Q.50 (4)

Q.51 (4)

Q.52 **(3)**

$\tau \;\alpha\; \theta \Rightarrow \tau = c\theta$

$dw = \tau\, d\theta = c\theta d\theta$

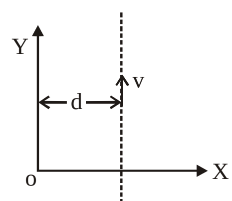
$\int dw = \int c\theta d\theta$

$w = \frac{1}{2}c\theta^2$

Q.53 (3)

$$\vec{\tau} = \frac{d\vec{L}}{dt} = \frac{4A_0 - A_0}{4} = \left(\frac{3A_0}{4}\right)$$

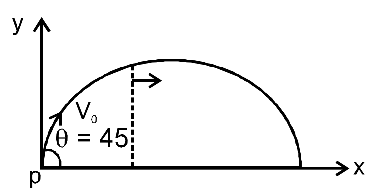
Q.54 (2)



⇒ L = (mvd) = constant

becouse v = const. and d = const.

Q.55 **(4)**



$x = v_0 \cos 45^o \times t = \frac{v_0 t}{\sqrt{2}}$

$\tau = mgx = \frac{mgv_0 t}{\sqrt{2}} = \frac{dL}{dt}$

$$\Rightarrow L = \frac{mgv_0}{\sqrt{2}} \int_0^{v_0/g} t\,dt = \frac{mv_0^3}{2\sqrt{2}g}$$

Q.56 (4)

$L = I\omega$

$\omega' = 2\omega$

$$\frac{1}{2}\left(\frac{1}{2}I\omega^2\right) = \frac{1}{2}I'\omega'^2$$

$$\frac{I\omega^2}{2} = I'\,4\omega^2$$

$$I' = \left(\frac{I}{8}\right)$$

$$L' = I'\omega' = \frac{I}{8}2\omega = \frac{I\omega}{4} = \left(L/4\right)$$

Q.57 **(2)**

No any external torque so L = constant;

$I_1\omega_1 = I_2\omega_2$

$(MR^2\omega) = (MR^2+2mR^2)\,\omega_2$

$$\Rightarrow \omega_2 = \left(\frac{M\omega}{M+2m}\right)$$

Q.58 (3)

external torque $\vec{\tau}_{ext} = 0$

$I_1\omega_1 = I_2\omega_2$

when he stretches his arms I

so $I_1 < I_2$

then $(\omega_1 > \omega_2)$

so, (L = constant)

Q.59 **(4)**

Torque

Q.60 (3)

$$\frac{1}{2}I\omega^2 = 10 \Rightarrow \frac{5}{2}\omega^2 = 10 \Rightarrow \omega = 2\,rad/sec$$

Angular Momentum

$L = I\omega = 5 \times 2 = 10$ joule-sec.

Q.61 (2)

$I_1\omega_1 = I_2\omega_2$

$MR_1^2\,\omega_1 = MR_2^2\,\omega_2$

$$\frac{R_1}{R_2} = \sqrt{\frac{\omega_2}{\omega_1}} = \frac{3}{1}$$

Q.62 **(2)**

Change in momentum = $2mV\cos\theta = \int F.dt$

$\therefore$ Change in angular momentum

$= \int Fd.dt = 2m\,Vd\cos\theta$

Q.63 (2)

$$KE = \frac{1}{2}I\omega^2 = \frac{1}{2}\times\left(\frac{\mu R}{2}\right)^2\frac{v^2}{R^2}$$

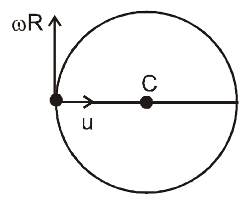
$$\frac{1}{4}Mv^2$$

$$\text{Total KE} = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$

$$\frac{1}{4}Mv^2 + \frac{1}{2}m\mu^2 \Rightarrow \frac{3}{4}mv^2$$

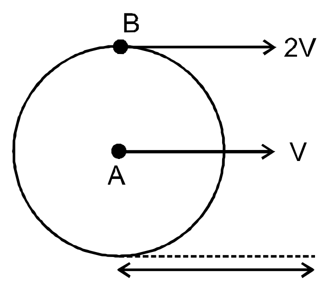
$$\text{Ratio} = \frac{1}{3}$$

Q.64 (3)



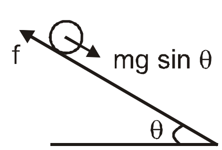
For pure rolling $\omega R = u$, $v = \sqrt{u^2 + (\omega R)^2} = \left(u\sqrt{2}\right)$

Q.65 (2)



When A point travels ℓ distance then B point 2ℓ so, 2ℓ length of string passes through the hand of the boy .

Q.66 (1)



$mg\sin\theta - f = ma$

$a = \left[\frac{mg\sin\theta - f}{m}\right]$

.......(i)

a is same for each body.

$f.R = I\alpha$

$\alpha = \frac{f.R}{mk^2}$

For solid sphere $k^2 = \frac{2}{5}R^2$ is minimum there fore α is maximum hence, k.E. for solid sphere will be max at bottom.

Q.67 (2)

$a = \left(\frac{g\sin\theta}{1+\frac{k^2}{R^2}}\right)$

For solid sphere $\Rightarrow \frac{k^2}{R^2} = \frac{2}{5}$

For hollow sphere $= \frac{2}{3}mR^2 = mk^2$

$\frac{k^2}{R^2} = \frac{2}{3}$

so $k_s < k_H$

then $a_s > a_H$

(so speed of solid sphere is greater then hollow sphere)

Q.68 **(1)**

$a = (g\tan\theta)$ so net force along the indined plane is zero so it will continue in pure rolling with constant angular velocity.

Q.69 **(4)**

There is no relative motion between sphere and plank so friction force is zero then no any change in motion of sphere and plank.

Q.70 (1)

Due to linear velocity body will move forward before pure rolling.

Q.71 (1)

Disk $= \frac{1}{2}I\omega^3 + \frac{1}{2}mv_1^2$

$= \frac{1}{2}\frac{MR^2}{2}\frac{v_1^2}{R^2} + \frac{1}{2}mv_1^2 \quad = \frac{3}{4}mv_1^2$

Ring $= \frac{1}{2}I\omega^2 + \frac{1}{2}mv_2^2 = \frac{1}{2}mr^2 \times \frac{v_2^2}{R^2} + \frac{1}{2}mv_2^2 = mv_2^2$

$\frac{1}{4}mv_1^2 = mv_2^2$

$\frac{v_1}{v_2} = \left(\frac{4}{3}\right)^{1/2}$

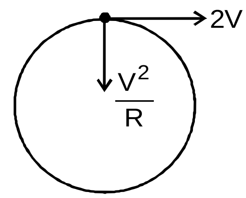
Q.72 **(4)**

$\because \mu = 0$

$a = g\sin\theta,\ t = \sqrt{\frac{2h}{g\sin\theta}}$

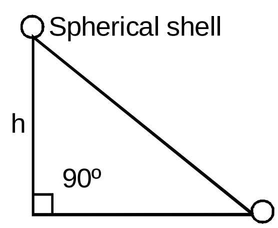
Q.73 **(3)**

$\frac{V^2}{R} = \frac{(2V)^2}{R'}$



$R' = 4R$

Q.74 **(3)**



$mgh = \frac{1}{2}Iw^2 + \frac{1}{2}mV^2,\ I_{shell} = \frac{2}{3}MR^2$

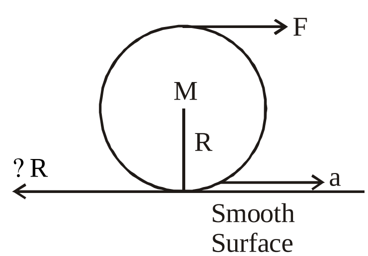
$mgh = \frac{1}{2} \times \frac{2}{3}MR^2 . \frac{V^2}{R^2} + \frac{1}{2}MV^2$

$mgh = \frac{1}{3}MV^2 + \frac{1}{2}mV^2$

$mgh = \frac{5}{6}mV^2 \Rightarrow V^2 = \frac{6gh}{5}$

Q.75 (1)

$\vec{F} = Ma$



$\Rightarrow \quad a = \dfrac{F}{M}$

For pure rolling

$a = \alpha R$

$F \times R = I\alpha$

$\alpha = \dfrac{FR}{I}$

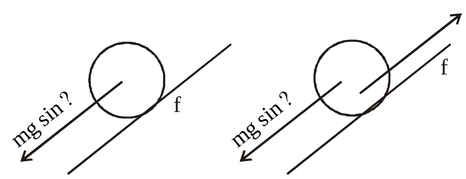
$\dfrac{F}{m} = \dfrac{FR.R}{I}$

$I = MR^2$

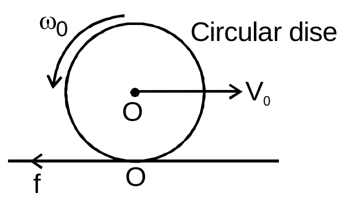
MR^2 is the moment of inertia of chin pipe.

Q.76 (4)

Q.77 (3)



Q.78 **(1)**



α is conserved about 0

$I\omega_0 - mVR = 0 \Rightarrow \qquad I\omega_0 = mVR$

$\dfrac{MR^2}{2}\omega_0 = mV_0R$

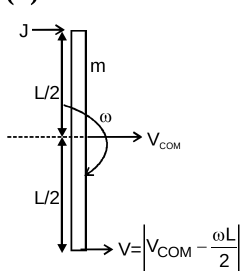
$\omega_0 = \dfrac{2V_0}{R} \Rightarrow \dfrac{V_0}{\omega_0 R} = \dfrac{1}{2}$

Q.79 (4)

As the inclined plane is smooth, the sphere can never roll rather it will just slip down.

Hence, the angular momentum remains conserved about any point on a line parallel to the inclined plane and passing through the centre of the ball.

Q.80 **(2)**



$J = MV_{COM} \Rightarrow V_{COM} = \dfrac{J}{M}$

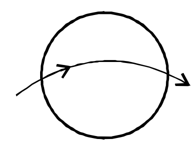
$J\dfrac{L}{2} = \dfrac{ML^2}{12}\omega$

$V = \left|\dfrac{J}{M} - \dfrac{6J}{ML}\dfrac{L}{2}\right|$

$J = \dfrac{MV}{2}$

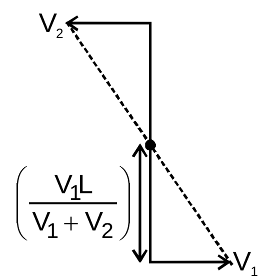
Q.81 **(4)**

(a) M is instantaneous axis of Rotation (I.A.R.) (b)



Magnitude is same but direction is different

Q.82 **(3)**



Q.83 **(1)**

Moment of inertia of disc $= \dfrac{mr^2}{2} = 0.5\ mr^2$

Moment of inertia solid sphere $= \dfrac{2}{5}\ mr^2$

Q.84 **(2)**

M.I. $= mr^2 = 4 \times 1^2 = 4$ kg m^2.

Q.85 **(4)**

$P = \tau\omega$

Q.86 **(4)**

$\vec{L} = m(\vec{r} \times \vec{V})$

$= 2 \times 2\,[(\hat{i} + \hat{j}) \times (\hat{i} - \hat{j} + \hat{k})]$

$= 4(-\hat{k} - \hat{j} - \hat{k} + \hat{i}) = 4(\hat{i} - \hat{j} - 2\hat{k})$

$\vec{L}$ = Angular Momentum along z-axis is the compoent of angular momentum along z-axis.

i.e. $= -8$ kg-m^2/sec

JEE-ADVANCED
OBJECTIVE QUESTIONS

Q.1 **(D)**

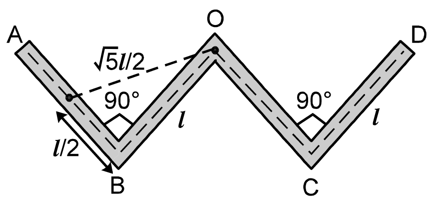
Given $a_A = 2\alpha = 5\ m/s^2$

$\Rightarrow \alpha = 5/2\ rad/s^2$

$\Rightarrow a_B = 1.(\alpha) = 5/2\ m/s^2$

Q.2 **(B)**

The given structure can be broken into 4 parts



For AB $I = I_{CM} + m \times d^2 = \dfrac{m\ell^2}{12} + \dfrac{5m}{4}\ell^2$;

$I_{AB} = \dfrac{4}{3}ml^2$

For BO $I = \dfrac{m\ell^2}{3}$

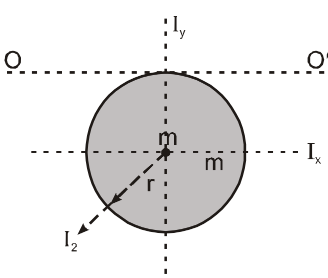
$\therefore$ For composite frame :(by symmetry)

$$I = 2[I_{AB} + I_{OB}] = 2\left[\frac{4m\ell^2}{3} + \frac{m\ell^2}{3}\right] = \frac{10}{3}ml^2.]$$

Q.3 **(D)**

Pearpendicular axis theorem

$$I_2 = I_x + I_y = \frac{mr^2}{2}$$



from symmetry $I_x = I_y$

$\Rightarrow I_x = \dfrac{mr^2}{4}$

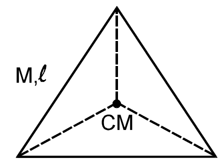
Perallel axis theorem

$I_{oo'} = I_x + mr^2$

$= \dfrac{mr^2}{4} + mr^2 = \dfrac{5}{4}mr^2$

Q.4 **(B)**

MI of the system w.r.t an axis ⊥ to plane & passing through one corner



$$= \frac{ML^2}{3} + \frac{ML^2}{3} + \left[\frac{ML^2}{12} + M\left(\frac{\sqrt{3}L}{2}\right)^2\right]$$

$$= \frac{2ML^2}{3} + \left[\frac{ML^2}{12} + \frac{3ML^2}{4}\right]$$

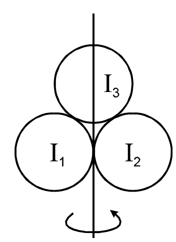
$$= \frac{2ML^2}{3} + \frac{10\ ML^2}{12} = \frac{3ML^2}{3} = \frac{18ML^2}{12}$$

$$= \frac{3}{2}ML^2$$

Now $\dfrac{3}{2}ML^2 = 3k^2$

$k = \dfrac{\ell}{\sqrt{2}}$ **[Ans.:** $\dfrac{\ell}{\sqrt{2}}$ **]**

Q.5 **(D)**



$I = I_1 + I_2 + I_3$

$I_1 = I_2 = \dfrac{3}{2}mr^2$

$I_3 = \dfrac{mr^2}{2}$

$\therefore I = I_1 + I_2 + I_3 = \dfrac{7}{2}mr^2$

Moment of inertia = $3mk^2$ where k is radius of gyration.

$3mk^2 = \dfrac{7}{2}mr^2 \Rightarrow k = \sqrt{\dfrac{7}{6}}\,r$

Q.6 **(D)**

Taking mass of plate $m = \dfrac{M}{6}$

Then MI of two plates through which the axis is passing = $\frac{ma^2}{6} \times 2 = \frac{ma^2}{3}$

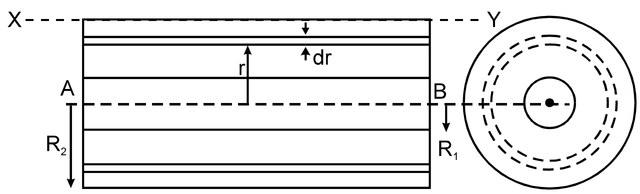
M.I of 4 plates having symmetrical position from the axis

$$= 4 \times \left[\frac{ma^2}{12} + m\left(\frac{a}{2}\right)^2\right] = 4 \times \left[\frac{ma^2}{3}\right]$$

$$\text{Total MI} = \frac{4ma^2}{3} + \frac{ma^2}{3} = \frac{5ma^2}{3}$$

using $\frac{M}{6} = m = MI = \frac{5Ma^2}{18}$

Q.7 **(D)**



Taking cylindrical element of radius r and thickness dr

$$dm = \frac{M}{\pi(R_2^2 - R_1^2)\ell} \times (2\pi r\, \ell\, dr)$$

$$I_{AB} = \int dI_{e\ell} = \int dm\, r^2$$

$$= \int_{R_1}^{R_2} \frac{2M}{(R_2^2 - R_1^2)} . r^3\, dr = \frac{1}{2}M(R_2^2 + R_1^2)$$

Using parallel axis theorem

$$I_{XY} = \frac{1}{2}M(R_2^2 + R_1^2) + MR_2^2 = \frac{M}{2}(3R_2^2 + R_1^2)$$

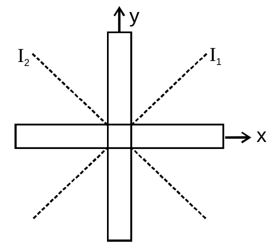
Q.8 **(A)**
By perpendicular axis theorem moment of inertia about any axis passing through centre and in the plane of plate will be I (by symmetry)

Q.9 **(B)**

Cylinder = $\frac{MR^2}{2}$, Square lamina = $\frac{MR^2}{6}$,

Solid sphere = $\frac{2}{5}$ MR^2

Q.10 **(C)**

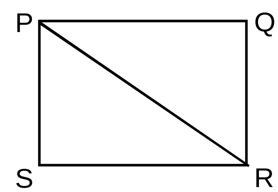


$$I_x = \frac{ML^2}{12} ; I_y = \frac{ML^2}{12}$$

$$I_x + I_y = I_z = I_1 + I_2$$

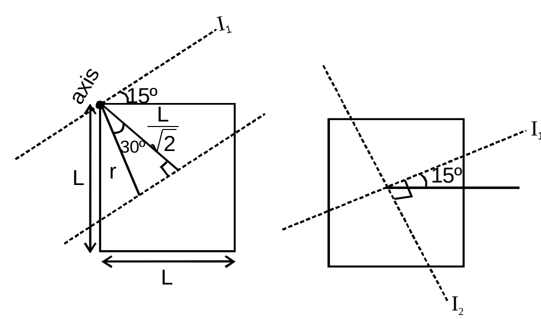
$$\frac{2.M.L^2}{12} = 2I_1 \Rightarrow I_1 = \frac{ML^2}{12}$$

Q.11 **(C)**



In case PQR
r is larger.

Q.12 **(B)**



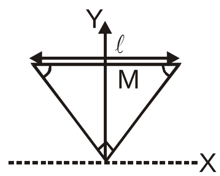
$$r = \frac{L}{\sqrt{2}} \cos 30^\circ \quad I_1 + I_2 = \frac{ML^2}{6}$$

$$= \frac{L\sqrt{3}}{2\sqrt{2}} \Rightarrow I_1 = \frac{ML^2}{12}$$

$$\therefore I' = \frac{ML^2}{12} + \frac{M.L^2.3}{8} = \frac{2ML^2 + 9ML^2}{24}$$

$$I' = \frac{11ML^2}{24}$$

The figure shows an isosceles triangular plate of mass M and base L. The angle at the apex is 90°. The apex lies at the origin and the base is parallel to X - axis.



Q.13 (C)

$\vec{\tau} = \vec{r} \times \vec{F}$

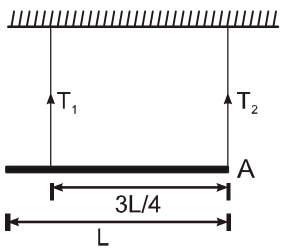
$= (-b\hat{i} - c\hat{k}) \times a\hat{j}$

$= (-b\hat{k} - c(-\hat{i}))$

$= -b\hat{k} + c\hat{i}$

Q.14 (C)

$\tau_A = 0$



$T_1 \times \frac{3L}{4} - mg\frac{L}{2} = 0$ ....(1)

$T_1 = \frac{2mg}{3}$

$T_1 + T_2 = mg$ ....(2)

$T_2 = \frac{mg}{3}$

$\frac{T_1}{T_2} = \frac{2}{1}$ **Ans.**

Q.15 (D)

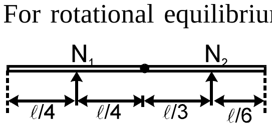
In equilibrium, torques of forces mg and Mg about an axis passing through O balance each other.

$mg.\frac{L}{2}\cos 30° = Mg\frac{L}{2}\cos 60°$

$\Rightarrow \frac{M}{m} = \sqrt{3}$

Q.16 (C)

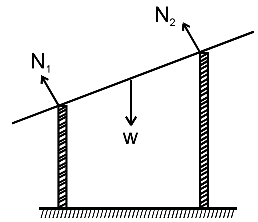
For rotational equilibrium



$N_1 \times \frac{\ell}{4} = N_2 \times \frac{\ell}{6}$

$N_1 : N_2 = 4 : 3$

Q.17 (C)



Balancing torque about the centre of the rod :

$N_1 \cdot \frac{\ell}{4} - N_2 \cdot \frac{\ell}{4} = 0 \Rightarrow N_1 = N_2.$

Q.18 (C)

$\vec{F}_{net} = (400-100)\hat{i} + (200+200)\hat{j} = 300\hat{i} + 400\hat{j}$

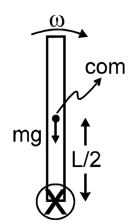
$\Rightarrow |\vec{F}| = 500$ N

Angle made by $\vec{F}_{net}$ with the vertical is $\theta = \tan^{-1}\left(\frac{300}{400}\right) = 37°$

also $\tau = 500$ R therefore point of application of the resultant force is at a distance R from the centre. Hence (C).

Q.19 (B)

For the circular motion of com :



$mg = m\left(\frac{L}{2}\right)\omega^2 \Rightarrow \omega = \sqrt{\frac{2g}{L}}$

Note : Since the reaction at the end is zero, the gravitational force will have to provide the required centripetal force.

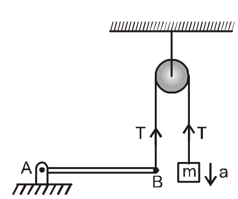
Q.20 (B)

Let α be the angular acceleration of rod and a be acceleration of block just after its release.

$\therefore$ $mg - T = ma$ ..... (1)

$T\ell - mg\frac{\ell}{2} = \frac{m\ell^2}{3}\alpha$ .... (2)

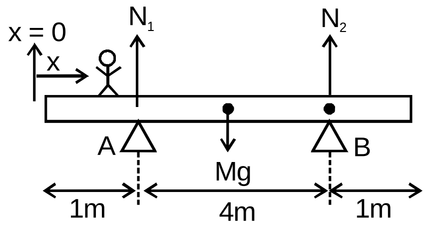
and $a = \ell\alpha$ .... (3)



Solving we get

$$T = \frac{5mg}{8} \quad \text{and} \quad \alpha = \frac{3g}{8\ell}$$

Q.21 **(B)**

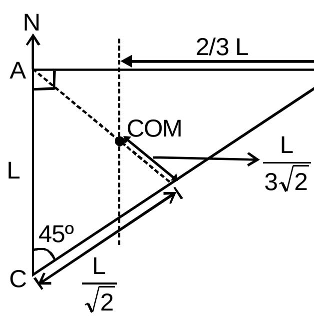


$N_1 + N_2 = (M + m) . g$

$\tau_B = 0$

$N_1 . 4 = Mg \times 2 + m (5 - x)$

Q.22 **(B)**



Net force = 0

T + N = Mg

....(1)

Net torque about B = 0

$\tau_B = 0$

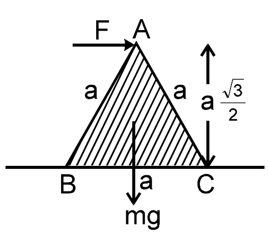
$N.L = Mg . \frac{2}{3} L,$ $\boxed{N = \frac{2}{3}Mg}$

Q.23 **(A)**

The tendency of rotating will be about the pont C. For minimum force, the torque of F about C has to be equal to the torque of mg about C.

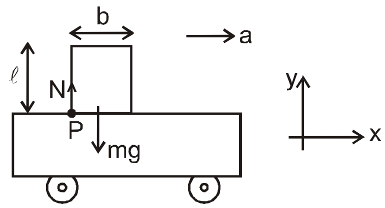
$$\therefore \quad F\left(a\frac{\sqrt{3}}{2}\right) = mg\left(\frac{a}{2}\right)$$

$$\Rightarrow \quad F = \frac{mg}{\sqrt{3}} \ \textbf{Ans.}$$



Q.24 **(B)**

For maximum a, normal reaction will shift to left most position.

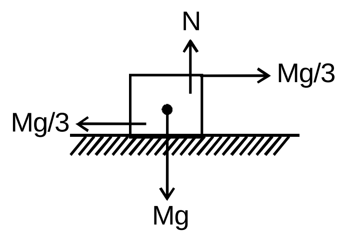


for rotational equilibrium τ_P = = 0 [in frame of truck]

$$ma\frac{\ell}{2} = mg\frac{b}{2} \qquad \Rightarrow a = \frac{gb}{\ell}$$

Q.25 **(B)**

$$f_{max} = \frac{1}{2} Mg$$



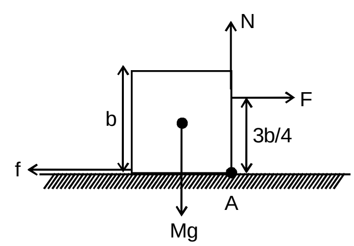
f = Mg/3

Torque Balance

$$\frac{Mg}{3} . \frac{a}{2} + \frac{Mg}{3} . \frac{a}{2} = N. x$$

$$\frac{Mga}{3} = mg\, x \Rightarrow x = \frac{a}{3}$$

Q.26 **(A)**



$f_{max} = \mu N$

....(1)

f = F

....(2)

$\tau_A = 0$

....(3)

$$F. \frac{3b}{4} = Mg.b/2$$

f = F = 2 Mg/3

$\therefore$ $f > \mu N \Rightarrow 2Mg/3 > \mu. Mg$

$$\mu > \frac{2}{3}$$

Q.27 **(C)**

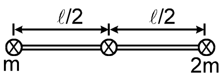
For (rod + particle) system :

$$\frac{1}{2}\left(\frac{m\ell^2}{3}\right)\left(\frac{v^2}{\ell^2}\right) + \frac{1}{2}mv^2 = 2\,mg\left(\frac{3\ell}{2}\right)$$

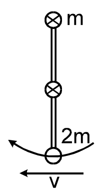
[Since, com will finally reach a height $2\left(\frac{3\ell}{4}\right)$]

$$\Rightarrow v = \sqrt{4.5g\ell}$$

Q.28 **(A)**

Decrease in PE = 

Increase in rotational K.E



$$\Rightarrow 2mg.\frac{\ell}{2} - mg.\frac{\ell}{2}$$

$$= \frac{1}{2}\,I.\omega^2 = \frac{1}{2}\left(2m\frac{\ell^2}{4} + m.\frac{\ell}{4}\right)\omega^2$$

$$\frac{mg\ell}{2} = \frac{1}{2}.\frac{3m\ell^2}{4}.\omega = \frac{3m\ell^2}{8}\omega^2$$

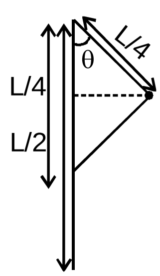
$$\omega = \sqrt{\frac{4g}{3\ell}} \text{ and } v = r\omega = \frac{\ell}{2}\sqrt{\frac{4g}{3\ell}} = \sqrt{\frac{g\ell}{3\ell}}$$

[Ans.: (a) V = $\sqrt{g\ell/3}$, $\omega = \sqrt{4g/3\ell}$]

Q.29 **(C)**

$$\omega = \sqrt{\frac{3g}{L}}$$

By Energy Conservation



$$\frac{1}{2}\,\frac{M}{2\times 3}\times\left(\frac{L}{2}\right)^2\times\frac{3g}{L}$$

$$= \frac{Mg}{2}\,\frac{L}{4}\,(1-\cos\theta)$$

$$\frac{ML^2}{4L}\times g = \frac{MgL}{2}\,(1-\cos\theta)$$

$$\cos\theta = \frac{1}{2} \Rightarrow \theta = 60^o$$

Q.30 **(D)**

$\vec{\tau} \times \vec{L}$

then

$\vec{\tau} \parallel \vec{L}$

so $\left(\vec{L}\right)$ may increase

Q.31 **(C)**

1st method : The direction of L is perpendicular to the line joining the bob to point C. Since this line keeps changing its orientation in space, direction of L keeps changing however as ω is constant, magnitude of L remain constant.

2nd method : The torque about point is perpendicular to the angular momentum vector about point C. Hence it can only change the direction of L, and not its magnitude.

Q.32 **(A)**

1st method : The angular momentum about axis CO is the component of angular momentum about point C along the line CO. This is constant both in direction and magnitude.

2nd method : Torque about axis CO is zero hence L about CO is constant in both direction and magnitude.

Q.33 **(D)**

Conserving the angular momentum : about the hinge

$$mua = \left[\frac{m(a^2+4a^2)}{12} + \frac{5}{4}ma^2\right]\omega$$

$$\Rightarrow \omega = \frac{3}{5}\,\frac{u}{a} \text{ Ans.}$$

Q.34 **(B)**

Since the work done is independent of the information about which point the rod is rotating, by work-energy theorem the kinetic energy will also be independent of the same.

Hence (B)

Q.35 **(A)**

By conservation of angular momentum about hinge O.

L = I ω

$$mv\frac{d}{2} = \left[\frac{Md^2}{12} + m\left(\frac{d}{2}\right)^2\right]\omega$$

$$\Rightarrow \frac{mvd}{2} = \left(\frac{md^2}{2} + \frac{md^2}{4}\right)\omega$$

$$\frac{mvd}{2} = \frac{3}{4}md^2\omega \quad \Rightarrow \frac{2}{3}\frac{v}{d} = \omega$$

Q.36 **(D)**

$$-\int T.dt = m.v - m \times 5$$
...(1)

$$\int T.dt \; . \; r = \frac{mr^2}{2} \; . \; \omega$$
...(2)

$$\omega = \frac{v}{r}$$
...(3)

$$\int T.dt = \frac{mv}{2}$$

$$5m - mv = \frac{mv}{2}, \; 5 = \frac{3v}{2}, \; v = \frac{10}{3} \frac{m}{sec}$$

Q.37 **(C)**

Angular momentum conservation

$MVR = (MR^2 + MR^2) . \omega$

$$\frac{V}{2R} = \omega$$

Q.38 **(A)**

$$\int \tau . \, dt = I\omega - 0$$

$$10 \times 1 = \frac{2\times(1)^2}{3} \times \omega \Rightarrow 15 \text{ rad/sec}$$

$\omega = 15$ rad/sec

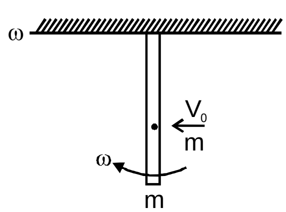
$$K.E. = \frac{1}{2} \times \frac{2\times(1)^2}{3} \times (15)^2 = 75 \text{ Joule}$$

Q.39 **(D)**

$(I + mR^2) . \omega = I\omega' + mvR$

$$\omega' = \frac{(I+mR^2).\omega - mVR}{I}$$

Q.40 **(A)**



$$\left[e = -\frac{(V_1 - V_2)}{u_1 - u_2}\right], \quad I = \frac{\frac{\omega L}{2} - 0}{V}$$

$$\Rightarrow \frac{\omega L}{2} = V$$

$$\frac{mVL}{2} = \frac{ML^2}{3} . \omega \Rightarrow \frac{mVL}{2} = \frac{ML^2}{3} \times \frac{2V}{L}$$

$$\Rightarrow \frac{M}{m} = \frac{3}{4}$$

Q.41 **(C)**

Immediately after string connected to end B is cut, the rod has tendency to rotate about point A.

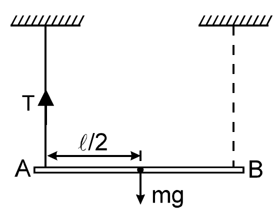
Torque on rod AB about axis passing through A and normal to plane of paper is

$$\frac{m\ell^2}{3}\alpha = mg\frac{\ell}{2} \quad \Rightarrow \alpha = \frac{3g}{2\ell}$$

Aliter : Applying Newton's law on center of mass

$mg - T = ma$
.....(i)

Writing $\tau = I\alpha$ about center of mass



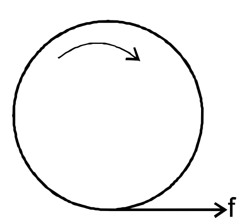
$$T\frac{\ell}{2} = \frac{m\ell^2}{12}\alpha \qquad ....(ii)$$

$$\text{Also } a = \frac{\ell}{2}\alpha \qquad ....(iii)$$

From (i) , (ii) and (iii)

$$\alpha = \frac{3g}{2\ell}$$

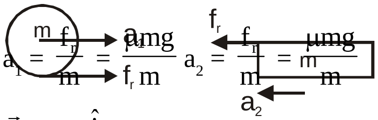
Q.42 **(C)**



Friction will at forward dir so body will always move in forward dir.

Q.43 **(D)**

FBD for sphere & block



$a_1 = \frac{f_r}{m} = \frac{\mu mg}{m}$ $a_2 = \frac{f_r}{m} = \frac{\mu mg}{m}$

$\vec{a}_1 = \mu g\hat{i}$

$\vec{a}_2 = -\mu g\hat{i}$

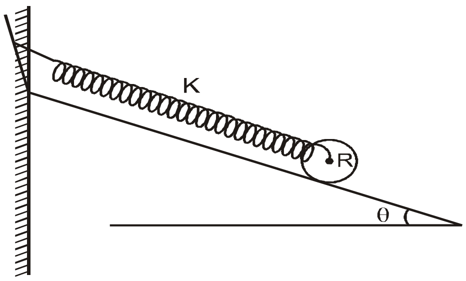
$\vec{a}_{rel} = \vec{a}_1 - \vec{a}_2 = 2\mu g\hat{i}$

$a_{rel} = 2\mu g.$

Q.44 **(C)**

Using Energy conservation,

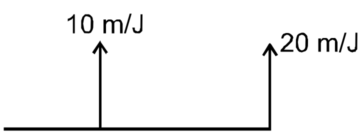
(at maximum distance V = 0 $V_0 = 0$)



$\frac{1}{2}Kx^2 = (mg\ x \sin\theta)$

$x = \left(\frac{2mg\sin\theta}{K}\right)$

Q.45 **(A)**



$20 = V_{cm} + wR$

$20 = 10 + \omega\left(\frac{\ell}{2}\right)$

$10 = \frac{\omega}{2}$

(ω = 20 rad / se)

Q.46 **(D)**

Since the two bodies have same mass and collide head-on elastically, the linear momentum gets interchanged.

Hence just after the collision 'B' will move with velocity 'v_0' and 'A' becomes stationary but continues to rotate at the same initial angular velocity $\left(\frac{v_0}{R}\right)$.

Hence, after collision.

$(K.E.)_B = \frac{1}{2}mv_0^2$

and $(K.E.)_A = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{2}{3}mR^2\right)\cdot\left(\frac{v_0}{R}\right)^2$

$\Rightarrow \frac{(K.E.)_B}{(K.E.)_A} = \frac{3}{2}$ Hence (D).

Note : Sphere 'B' will not rotate, because there is no torque on 'B' during the collision as the collision is head-on.

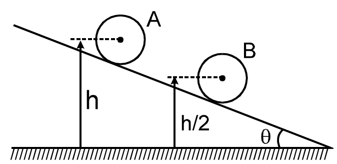
Q.47 **(B)**

Disc in pure rolling and external force zero after smooth surface pure rolling continue.

Q.48 **(A)**

Just before collision Between two Balls

potential energy lost by Ball A = kinetic energy gained by Ball A.



$mg\frac{h}{2} = \frac{1}{2}I_{cm}\omega^2 + \frac{1}{2}mv_{cm}^2$

$= \frac{1}{2} \times \frac{2}{5}mR^2 \times \left(\frac{v_{cm}}{R}\right)^2 + \frac{1}{2}mv_{cm}^2$

$= \frac{1}{5}\ mv_{cm}^2 + \frac{1}{2}\ mv_{cm}^2$

$\Rightarrow \frac{5}{7}mgh = mv_{cm}^2 \Rightarrow \frac{mgh}{7} = \frac{1}{5}\ mv_{cm}^2$

After collision only translational kinetic energy is transfered to ball B

So just after collision rotational kinetic energy of Ball A $= \frac{1}{5}\ mv_{cm}^2 = \frac{mgh}{7}$

Q.49 **(C)**

Let velocity of c.m. of sphere be v. The velocity of the plank = 2v.

Kinetic energy of plank $= \frac{1}{2} \times m \times (2v)^2$

$= 2mv^2$

Kinetic energy of cylinder

$= \frac{1}{2}mv^2 + \frac{1}{2} + \left(\frac{1}{2}mR^2\omega^2\right)$

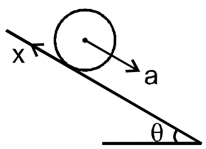
$= \frac{1}{2}mv^2\left(1 + \frac{1}{2}\right) = \frac{3}{2}.\frac{1}{2}mv^2$

$$\therefore \quad \frac{\text{K.E. of plank}}{\text{K.E. of sphere}} = \frac{2mv^2}{\frac{3}{4}mv^2} = \frac{8}{3}.$$

Q.50 **(C)**

The horizontal shift of end x will be double the shift of centre of spool. Hence centre travels by $\frac{S}{2}$.

Q.51 **(D)**

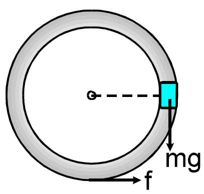


Torque about COM

f.R = I · α (a = αR)

$$f.R = \frac{mR^2}{2}\ a = \left(\frac{mR^2}{2}\cdot R\right) \Rightarrow \left(f = \frac{ma}{2}\right)$$

Q.52 **(B)**



f = 4 ma .... (1)

$(mg - f)r = (3\,mr^2 + mr^2)\,\alpha$

mg – f = 4 ma .... (2)

from (1) and (2)

$\Rightarrow$ 8 ma = mg

$$\Rightarrow a = \frac{g}{8} \Rightarrow \alpha = \frac{g}{8r}$$

Q.53 **(B)**

Here, $u = V_0$, $\omega_0 = -\frac{V_0}{2R}$

At pure rolling ;

$$V = V_0 - \left(\frac{F_f}{m}\right)t$$

$$\& \quad \frac{V}{R} = -\frac{V_0}{2R} + \left(\frac{F_f}{m.R}\right)t \text{ (In pure rolling } V = R\omega)$$

$$\left(\alpha = \frac{\tau}{I} = \frac{F_f.R}{mR^2}\right)$$

$$\Rightarrow V_0 - V = V + \frac{V_0}{2}$$

$$\Rightarrow 2V = \frac{V_0}{2} \Rightarrow V = \frac{V_0}{4}$$ **Ans.**

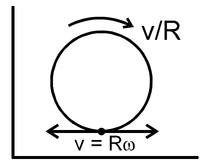
Q.54 **(D)**

As the disc is in combined rotation and translation, each point has a tangential velocity and a linear velocity in the forward direction.

From figure

v_{net} (for lowest point = v – Rω = v – v = 0.

$$\text{and Acceleration} = \frac{v^2}{R} + 0 = \frac{v^2}{R}$$



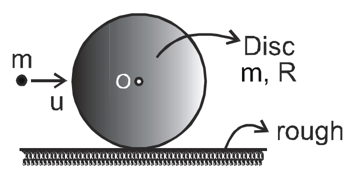
(Since linear speed is constant

Hence (D).

Q.55 **(A)**

Angluar momentum conservation about contact point

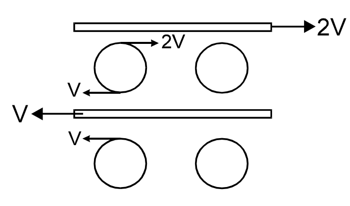
$muR = (I_A)\omega$



$$I_A = \left(\frac{mR^2}{2} + mR^2\right) + m\left(\sqrt{2}R\right)^2 = \frac{7}{2}mR^2$$

$$\omega = \frac{muR}{\frac{7}{2}mR^2} = \frac{2u}{7R}$$ **Ans.**

Q.56 **(A)**



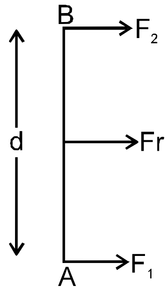
Since there is no slipping at any interface, the velocities of bottom and upper most point of lower and upper cylinder are shown in figure.

$$\text{Angular velocity of upper cylinder} = \frac{2V + V}{2R} = \frac{3V}{2R}$$

$$\text{Angular velocity of lower cylinder} = \frac{V - 0}{2R} = \frac{V}{2R}$$

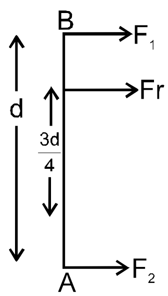
The ratio is $\frac{3}{1}$

Q.57 **(D)**



Torque about point A

$T_A = Fr\left(d/2\right) + F_2(d)$



$\tau_A^1 = Fr\left(\frac{3d}{4}\right) + F_1(d)$

$(F_1 + F_2)\frac{d}{2} + F_2 d = (F_1 + F_2)\left(\frac{3d}{4}\right) + F_1 d$

$\frac{F_1 + F_2}{2} + F_2 = \left(\frac{3}{4}F_1 + \frac{3}{4}F_2 + F_1\right)$

$\frac{F_1}{2} - \frac{3}{4}F_1 - F_1 = \left(\frac{3}{4}F_1 + F_2 - \frac{F_2}{2}\right)$

$\left(\frac{-F_1}{4} - F_1\right) = \left(\frac{-F_2}{4} - \frac{F_2}{2}\right)$

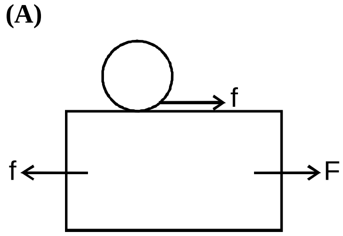
$\frac{5F_1}{4} = \frac{3F_2}{4}$

$5F_1 = 3F_2$

$\frac{F_1}{F_2} = \left(\frac{3}{5}\right)$.

Q.58 **(C)**

Q.59 **(A)**



Q.60 (D)

Due to torque of friction about CM ω eventually decreases to zero, initially there is no translation. Friction is sufficient for puse rolling therefore after sometime pure rolling beging. There is no external force in × direction therefore momentum is conserved along × direction.

Q.61 **(D)**

$a = \frac{5g\sin\theta}{7}$

(i) $25 = \frac{1}{2}\,a\,t^2_{Q\text{ to }0}$ ...(1)

$5 = \frac{1}{2}\,a\,t^2_{P\text{ to }0}$

...(2)

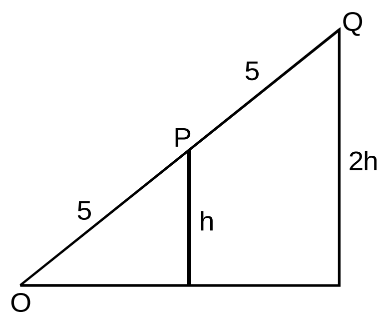
$t_{Q\text{ to }0} = \sqrt{\frac{45}{a}}$

$t_{P\text{ to }0} = \sqrt{\frac{25}{a}}$

(ii) $Mg\sin\theta - f = ma$

$fR = I\alpha$

$mg\sin\theta - \frac{I\alpha}{R} = ma$



$mg\sin\theta = I\frac{\alpha}{R} + ma$ $\boxed{a = \alpha R}$

$mg\sin\theta = \frac{Ia}{R^2} + ma$

$mg\sin\theta = a\left(\frac{I}{R^2} + m\right)$

$a = \frac{mg\sin\theta}{m\left(1 + \frac{I}{mR^2}\right)}$ $I = \frac{2}{5}mR^2$

$a = \frac{g\sin\theta}{1 + \frac{2}{5}\frac{MR^2}{MR^2}}$

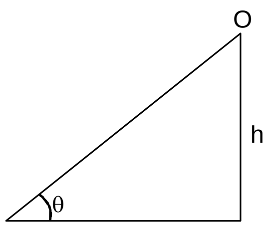
$$a = \frac{g\sin\theta}{1+\frac{2}{5}} = \frac{5g\sin\theta}{7}$$

(iii) $K.E_{\text{at O from P}} = mgh$

$K.E_{\text{at O from P}} = 2\,mgh$

Q.62 **(C)**

$$mgh = \frac{1}{2}\,I\omega^2 + \frac{1}{2}\,mV^2$$



$$mgh = \frac{1}{2}\left(mR^2 \times \frac{V^2}{R^2}\right) + \frac{1}{2}\,mV^2$$

$2mgh = mV^2\,(1 + C)$

$$V^2 = \frac{2gh}{1+C}$$

$$K.E. = \frac{1}{2}\,mV^2 = \frac{1}{2}\,m\,.\,\frac{2gh}{1+C} = \frac{mgh}{1+C}$$

$$K.E_{ring} = \frac{mgh}{1+1} = \frac{mgh}{2}$$

$$K.E_{coin} = \frac{mgh}{1+\frac{1}{2}} = \frac{2}{3}\,mgh$$

$$K.E_{\text{solid sphere}} = \frac{mgh}{1+\frac{2}{5}} = \frac{5}{7}\,mgh$$

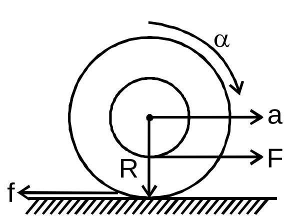
$$\text{Ratio} = \frac{1}{2} : \frac{2}{3} : \frac{5}{7}$$

$= 21 : 28 : 30$

Q.63 **(B)**

$$\alpha = \frac{a}{R}$$

$F - f = Ma$

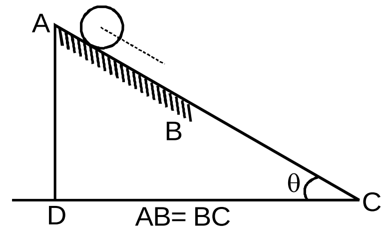


$f\,.\,R - F.r = I.\,\alpha$

assumed direction of friction is same so spool rotates clockwise and thread winds.

Q.64 **(B)**

For pure rolling



$Mg\sin\theta - f = Ma$

$$f\,.\,R = \frac{MR^2}{2}\,.\,\frac{a}{R}$$

$$Mg\sin\theta = \frac{3Ma}{2} \Rightarrow a = \frac{2g\sin\theta}{3}$$

$$\text{Now } \frac{L}{2} = \frac{1}{2}\,.\,\frac{2g\sin\theta.t^2}{3}$$

$$V_B{}^2 = \frac{2\times 2g\sin\theta}{3} \times \frac{L}{2} = \frac{2g\sin\theta L}{3}$$

$$\omega_B = \frac{V_B}{R}$$

$$\therefore V_C{}^2 - V_B{}^2 = 2g\sin\theta\,.\,\frac{L}{2} = gL\sin\theta$$

$$V_C{}^2 = g\sin\theta\,L + \frac{2gL\sin\theta}{3}$$

$$= \frac{5gL\sin\theta}{3}$$

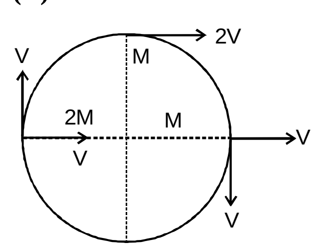
$$\therefore \text{Rotational kinetic energy} = \frac{1}{2}\,.\,\frac{MR^2}{2}\,.\,\frac{V_B^2}{R^2}$$

$$= \frac{1}{2}\,.\,\frac{M.2g\sin\theta L}{3}$$

Q.65 **(D)**

Q.66 **(D)**

Q.67 **(A)**

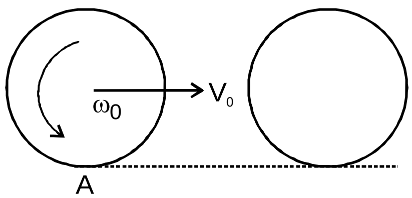


$= \frac{1}{2} MV^2 + \frac{1}{2} \frac{MR^2 \times V^2}{R^2} + \frac{1}{2} M(2V^2) + \frac{1}{2} M(2V)^2 + \frac{1}{2} . 2M (2V)^2$

$= 6MV^2$

Q.68 **(C)**

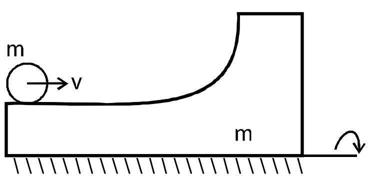
Angular momentum conservation (about A)



$\frac{2}{5} MR^2\omega_0 = MV_0R$ $\boxed{5V_0 = 2\omega_0 R}$

Q.69 **(B)**

When ball at maximum height block and ball has equal velocity So Using momentum conservation



$P_i = mv$

$P_f = 2mv_0$(v_0 final velocity)

$P_i = P_t$

$mv = 2\, mv_0$

$V_0 = \left(\frac{V}{2}\right)$

Using energy conservation

$\frac{1}{2} I\omega^2 + \frac{1}{2} mv^2 = \frac{1}{2} I\omega^2 + \frac{1}{2} 2mv_0^2 + mgh$

$(I = mR^2)$

$v = \omega R$

$\frac{1}{2} mv^2 = \frac{1}{2} 2mv_0^2 + 2mgh$

$v^2 - 2\frac{v^2}{4} = 2gh$

$\left(h = \frac{v^2}{4g}\right)$

Q.70 **(D)**

As torque = change in angular momentum

$\therefore$ $F . \Delta t = mv$ (Linear) ..... (1)

and $\left(F \cdot \frac{\ell}{2}\right) \Delta t = \frac{m\ell^2}{12} \cdot \omega$ (Angular) ..... (2)

Dividing : (1) and (2)

$2 = \frac{12v}{\omega\ell} \Rightarrow \omega = \frac{6v}{\ell}$

Using S = ut :

Displacement of COM is $\frac{\pi}{2} = \omega t = \left(\frac{6v}{\ell}\right) t$

and $x = vt$

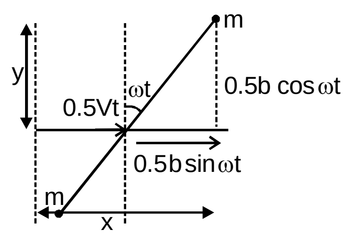
Dividing $\frac{2x}{\pi} = \frac{\ell}{6}$

$\Rightarrow$ $x = \frac{\pi\ell}{12} \Rightarrow$ Coordinate of A will be $\left[\frac{\pi\ell}{12} + \frac{\ell}{2}, 0\right]$

Hence (D).

Q.71 **(C)**

Angular Momentum conservation about C.O.M.



$2m.v.\frac{b}{2} + mv\frac{b}{2} = \left(2m.\frac{b^2}{4}.\omega\right) + 0$

$\Rightarrow \frac{3mvb}{2} = \frac{mb^2}{2}.\omega$ $\boxed{\omega = \frac{3V}{b}}$

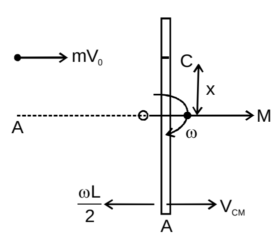
L.M.C. $2mV - mV = 2mV'$

$V' = 0.5\, V$

$x = 0.5Vt + 0.5b \sin \omega t$

$y = 0.5 \cos\omega t$ where $\omega = \frac{3V}{b}$

Q.72 **(B)**

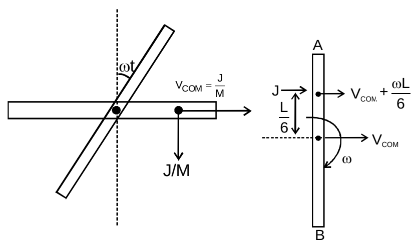


L.M.C. $mV_0 = MV_{CM}$
A.M.C. (A)

$$mv_0 x = \frac{ML^2}{12}\omega$$

$$\frac{\omega L}{2} = V_{CM} \Rightarrow \boxed{x = \frac{L}{6}}$$

Q.73 **(D)**
$J = M.V_{COM}$



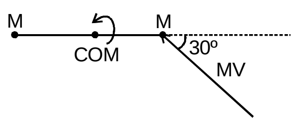
$$\Rightarrow V_{COM} = \frac{J}{M}$$

$$\frac{J.L}{2} = \frac{ML^2\omega}{12}$$

$$\Rightarrow \omega = \frac{6J}{ML}$$

$$\theta = \omega t = \frac{6J}{ML} \cdot \frac{\pi ML}{12J} = \frac{\pi}{2} \Rightarrow \boxed{V = \frac{\sqrt{2}J}{M}}$$

Q.74 **(C)**

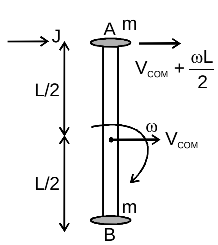


$\int \vec{L}dt =$ change in angular momentum

$$MV\sin 30^\circ \frac{L}{2} = \frac{2ML^2\omega}{4} \quad \boxed{\frac{V}{2L} = \omega}$$

Q.75 **(B)**

$$J.\frac{L}{2} = I\omega$$



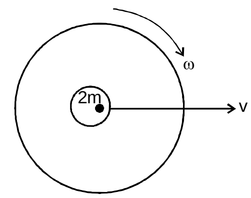
$$J.\frac{L}{2} = 2\frac{ML^2}{4}.\omega \Rightarrow \omega = \frac{J}{ML}$$

$J = 2M\,V_{COM}$

$$V_{COM} = \frac{J}{2M}$$

Now, $V_A = \frac{J}{2M} + \frac{J}{ML} \cdot \frac{L}{2} = \frac{J}{M}$

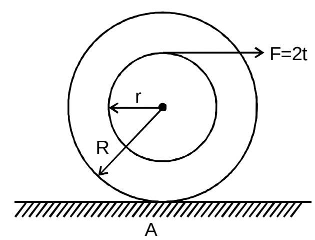
Q.76 **(C)**



$$\frac{2}{3} mR^2\omega + mvR + mvR = \frac{8}{3} mvR$$

Water is at rest w.r.t centre.

Q.77 **(C)**



$2t - f = 0$
$\tau_A = wt\,(R + r)$

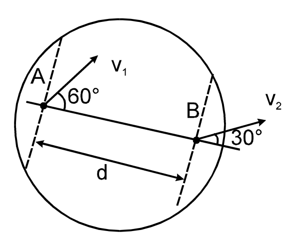
$$\int_0^L dL = \int_0^t 2t(R+r)dt$$

$L = (R + r)\,t^2$

Q.78 **(D)**
For rigid body separation between two point remains same.

$v_1 \cos 60° = v_2 \cos 30°$

$$\frac{v_1}{2} = \frac{\sqrt{3}\,v_2}{2} \Rightarrow v_1 = \sqrt{3}\,v_2$$

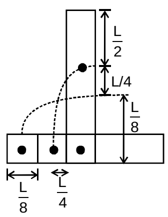


$$\omega_{disc} = \left|\frac{v_2\sin 30° - v_1\sin 60°}{d}\right| = \left|\frac{\frac{v_2}{2} - \frac{\sqrt{3}v_1}{2}}{d}\right|$$

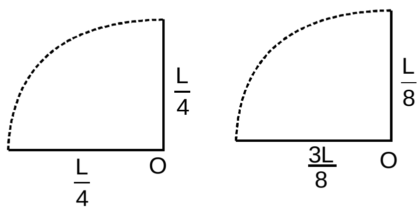
$$= \left|\frac{v_2 - \sqrt{3}\times\sqrt{3}v_2}{2d}\right| = \frac{2v_2}{2d} = \frac{v_2}{d}$$

$$\omega_{disc} = \frac{v_2}{d}$$

Q.79 **(B)**

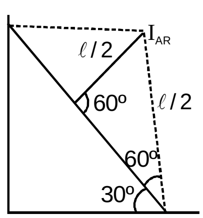


There is no force in Horizontal direction C.O.M. will remain constant



Quarter circle with Radius $\frac{L}{4}$ It is not circle

Q.80 **(C)**



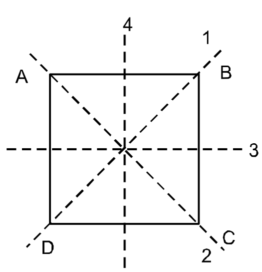
$$\omega = \frac{V}{\ell\sin\theta} = \frac{2v}{\ell}$$

JEE-ADVANCED

MCQ/COMPREHENSION/COLUMN MATCHING

Q.1 (A, B, C)
Sphere is rotating about a diameter
so $a = \alpha R$
but, R is zero for particles on the diameter.

Q.2 (A, B, C)



Using perpendicular theorem

$I_0 = I_4 + I_3$ $\quad$ $I_3 = I_4$
$I_0 = I_1 + I_2$ $\quad$ $I_2 = I_1$
$I_3 = I_2$
so, $(I_0 = I_1 + I_3)$

Q.3 (A,B,C,D)
$I_1 + I_3 = I \Rightarrow 2I_1 = 2I_3 = I$
$I_2 + I_4 = I,\ 2I_2 = 2I_4 = I$
$I_1 = I_2 = I_3 = I_4 = I$

Q.4 (A, D)

(1) For no slipping
$\mu mg\cos\theta \ge mg\sin\theta$ ....(1)
For toppling

$$mg\sin\theta\ \frac{h}{2} \ge mg\cos\theta.\ \frac{a}{2}$$

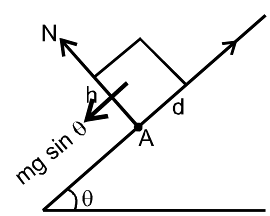
.........(2)
for minimum μ (by dividing)

$$\mu.\frac{2}{a} = \frac{2}{h}$$

$$\mu_{min} = \frac{a}{h}.$$

[Ans.: a/h]

(2)



If $f > mg\sin\theta$
$\mu mg\cos\theta > mg\sin\theta$
$(\mu > \tan\theta)$ block will topple before sliding
torque about point A $\tau_A = 0$

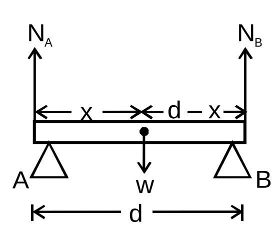
$$mg\sin\theta\left(\frac{h}{2}\right) = mg\cos\theta\ \frac{a}{2}$$

$$\tan\theta = \left(\frac{a}{h}\right)$$

$$\mu > \left(\frac{a}{h}\right)$$

If $\mu > \tan\theta$ (block will slide)

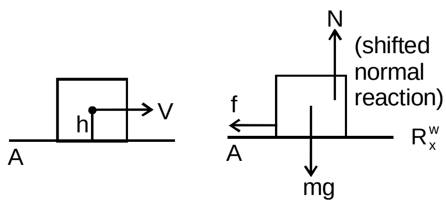
Q.5 **(B, C)**



$N_A + N_B = W$
$W(d - x) = N_A \cdot d$

Q.6 (B, C, D)
Body is in equilibrium
So $\tau_{net} = 0$
or
$F_{net} = 0$

Q.7 **(A, B, D)**

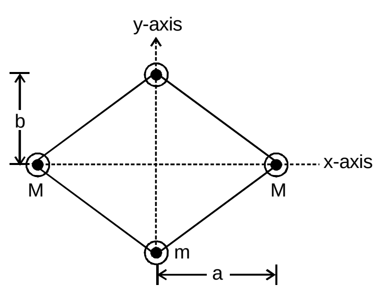


Angular momentum is not conserved

Q.8 (A, B, C)

(A) $KE = \frac{1}{2} I\omega^2$

I depends on m
$\therefore$ KE depends on m



(B) $I_{y_{axis}} = 2Ma^2$

$K.E. = \frac{1}{2} \times 2Ma^2\omega^2 = Ma^2\omega^2$

$I_{z_{axis}} = 2(Ma^2 + mb^2)$

$K.E. = \frac{1}{2} I\omega^2 = (Ma^2 + mb^2)\omega^2$

Q.9 (A, B)
In absence of external force linear momentum and angular momentum remains const.

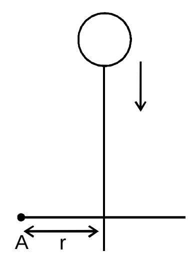
Q.10 (B, C)
External force will act at hinge so linear momentum of system will not remain const. but torque of external force is zero about hinge so $\vec{L}$ = const., collision is elastic so K.E = const.

Q.11 (A, B, D)
at the moment when ring is placed friction will act between them due to relative motion. Friction is internal force between them so angular momentum of system is conserved.
$I_1\omega_1 = I_2\omega_2$

$$\frac{mR^2}{2}\omega_0 = \left(\frac{mR^2}{2} + mR^2\right)\omega$$

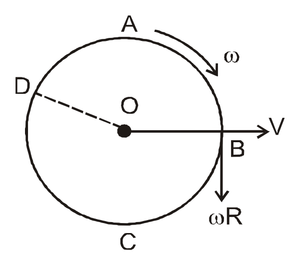
$$\omega = \frac{\omega_0}{3}$$

Q.12 (A,C,D)
τ is constant(mgr)



Q.13 (A,C,D)

Q.14 (A, C, D)



for pure rolling
$V = \omega R$
$V_A = 0$
$V_B = \sqrt{2}\,V$
$(V_C = 2V)$

Q.15 (A, C)
If bicycle is accelerating on a horizontal plane then friction on front wheel will be backward and on rear wheel it will be in forward direction.
But if bicycle is accelerating down an inclined plane then friction on rear wheel may be backward or forward both.

Q.16 (B,C,D)

After B there is no friction

$\therefore$ $F_{net} \uparrow$ or acceleration $\uparrow$

F – f= ma

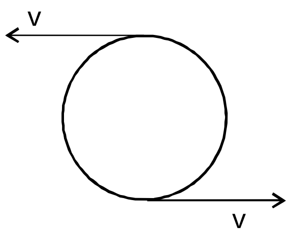
$$f \cdot R = \frac{mR^2}{2} \cdot \frac{a}{R}$$

$f = \frac{ma}{2}$ acceleration became doulble

Q.17 (A,C,D)

Q.18 (B, C)

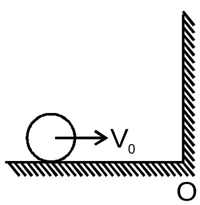
Velocity of COM is zero



$\omega = \frac{v}{R}$

Q.19 (A,B,D)

(A) Change in Angular Mom.



$= L_f - L_i$

$= (I\omega - mV_0R) - (I\omega + mV_0R)$

$= -2mV_0R$

(B) Impulse = Change in momentum

$= -2mV_0R$

Q.20 (C, D)

All points in the body, in plane perpendicular to the axis of rotation revolve in concentric circles. All points lying on circle of same radius have same speed (and also same magnitude of acceleration) but different directions of velocity (also different directions of acceleration)

Hence there cannot be two points in the given plane with same velocity or with same acceleration.

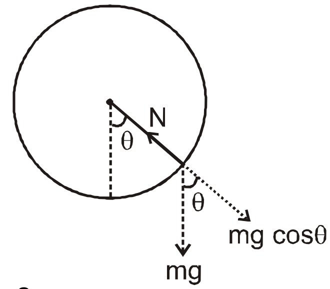
As mentioned above, points lying on circle of same radius have same speed.

Angular speed of body at any instant w.r.t. any point on body is same by definition.

Q.21 (A, B, C)

By angular momentum conservation ;

$$L = I\,\omega \Rightarrow mv\frac{R}{2} + mvR = 2mR^2\omega$$



$$\frac{3}{2}mvR = 2mR^2\omega$$

$$\omega = \frac{3v}{4R}$$

Also as the time of contact ;

$$mg\cos\theta - N = \frac{mv^2}{R}$$

$$\therefore N = mg\cos\theta - \frac{mv^2}{R}$$

when it ascends θ decreases so cosq increases and v decreases.

$\therefore$ mgcosθ is increasing and $\frac{mv^2}{R}$ is decreasing

$\therefore$ we can say N increases as wheel ascends.

Q.22 (B)

Let the angular speed of disc when the balls reach the end be ω. From conservation of angular momentum

$$\frac{1}{2}mR^2\omega_0 = \frac{1}{2}mR^2\omega + \frac{m}{2}R^2\omega + \frac{m}{2}R^2\omega$$

or $\omega = \frac{\omega_0}{3}$

Q.23 (C)

The angular speed of the disc just after the balls leave the disc is

$$\omega = \frac{\omega_0}{3}$$

Let the speed of each ball just after they leave the disc be v.

From conservation of energy

$$\frac{1}{2}\left(\frac{1}{2}mR^2\right)\omega_0^2$$

$$= \frac{1}{2}\left(\frac{1}{2}mR^2\right)\omega^2 + \frac{1}{2}\left(\frac{m}{2}\right)v^2 + \frac{1}{2}\left(\frac{m}{2}\right)v^2$$

solving we get

$v = \dfrac{2R\omega_0}{3}$

NOTE : $v = \sqrt{(\omega R)^2 + v_r^2}$; v_r = radial velocity of the ball

Q.24 (D)

Workdone by all forces equal

$K_f - K_i = \dfrac{1}{2}\left(\dfrac{m}{2}\right)v^2 = \dfrac{mR^2\omega_0^2}{9}$

Q.25 (D)

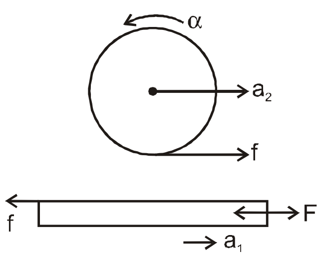
Q.26 (C)

Q.27 (A)
The free body diagram of plank and disc is
Applying Newton's second law

$F - f = Ma_1$ .... (1)

$f = Ma_2$ .... (2)

$FR = \dfrac{1}{2}MR^2\alpha$ .... (3)



from equation 2 and 3

$a_2 = \dfrac{R\alpha}{2}$

From constraint $a_1 = a_2 + R\alpha$

$\therefore a_1 = 3a_2$ .... (4)

Solving we get $a_1 = \dfrac{3F}{4M}$ and $\alpha = \dfrac{F}{2MR}$

If sphere moves by x the plank moves by L + x. The from equation (4)

$L + x = 3x$ or $x = \dfrac{L}{2}$

Q.28 (C)

Q.29 (A)
Q.30 (C)
Q.31 (C)

Q.32 (B)
By conservation of any momentum,
$I_\omega w_\omega + I_m\omega_m$ = constant.]

Q.33 (C)

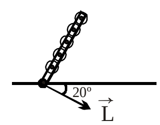
$T = \dfrac{2u\sin\theta}{g} = 3.6$ sec.

Q.34 (A)

$0.4 \times \dfrac{300}{0.3} = 0.4 \times 160 + 20\,\omega$

$40 = 64 + 20\,\omega$
$\omega = -1.2$ rad/s
$\theta = \omega t = 1.2 \times 3.6 = 4.32$ rad

Q.35 (B)



Direction of velocity of all points on the rod is perpendicular to the plane outwards

Q.36 (B)
As angular velocity is uniform so angular acceleration is zero which means there should be no torque in vertical direction]

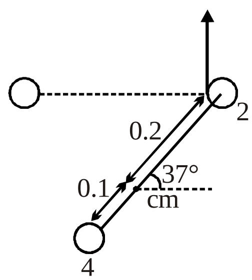
Q.37 (A) p,q,r (B) p,q,r (C) p,q (D) p,q,r
Since all forces on disc pass through point of contact with horizontal surface, the angular momentum of disc about point on ground in contact with disc is conserved. Also the angular momentum of disc in all cases is conserved about any point on the line passing through point of contact and parallel to velocity of centre of mass.
The K.E. of disc is decreased in all cases due to work done by friction.
From calculation of velocity of lowest point on disc, the direction of friction in case A, B and D is towards left and in case C is towards right.
The direction of frictional force cannot change in any given case.

Q.38 (A) p (B) q,s (C) p (D) q,s
(A) Speed of point P changes with time
(B) Acceleration of point P is equal to ω^2x (ω = angular speed of disc and x = OP). The acceleration is directed from P towards O.
(C) The angle between acceleration of P (constant in magnitude) and velocity of P changes with time. Therefore, tangential acceleration of P changes with time.
(D) The acceleration of lowest point is directed towards centre of disc and remains constant with time

NUMERICAL VALUE BASED

Q.1 [2]

CM at 0.1



$L_i = L_f$

$2 \times 0.5 \times 0.2 \sin 37°$

$= -1 \times 0.5 \times 0.2 \cos 37° + \frac{2\times4}{2+4} \times (0.3)^2\ \omega$

$0.2 \times \frac{3}{5} + 0.1 \times \frac{4}{5} = \frac{8}{6} \times 0.3^2\ \omega$

$\omega = \frac{5}{3} = 1.67$ rad/s $\simeq 2$ rad/s

Q.2 [7]

Conservation of angular momentum about the fixed axis.

$4600 \times 1 + 1 \times 80 \times 5 = (4600 + 80 \times 1^2) \times \omega_f$

$\omega_f = 32/468$

$\Delta\omega = 0.068$

$6.8 \times 10^{-2} = 7 \times 10^{-2}$

Q.3 [5625]

$\frac{1}{2} \times 15 \times 2^2 \times 2 = \left(\frac{1}{2}\times15\times2^2 + 2\times1^2\right)\omega$

$\frac{60}{32} = \omega \Rightarrow \omega = \frac{15}{8}$ rad/s

$K = \frac{1}{2} \times 32 \times \frac{225}{64} = \frac{225}{4}$ J $= 56.25$ J

Q.4 [7]

Angular momentum conservation

$$2\left(Mv\frac{d}{2}\right) = \left[\frac{Md^2}{12} + 2\left(M\left(\frac{d}{2}\right)^2\right)\right]\omega$$

$\Rightarrow \omega = \frac{12v}{7d}$

$KE_i = 2\left(\frac{1}{2}Mv^2\right)$

$\Rightarrow \quad KE_f = \frac{1}{2} I\omega^2 = \frac{1}{2}\left(\frac{7Md^2}{12}\right)\omega^2$

$\Delta KE = \frac{1}{7}Mv^2 \quad \Rightarrow \frac{KE_i}{\Delta KE} = 7$

Q.5 [5]

$L_{final} = 0$

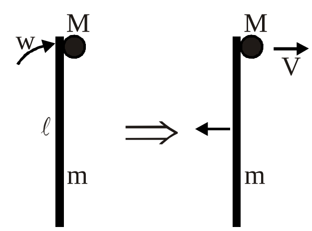
$L_{initial} = \frac{2}{5}mR^2\omega_0 - mv_0R$

$\omega_0 = \frac{5v_0}{2R}$ or

$\omega_0 = \frac{5\times v_0}{2\times(0.05)} = 5$ rad/sec.

Q.6 [12]

cons. of E $\Rightarrow \frac{1}{2}\left(\frac{1}{12}m\ell^2\right)w^2 = \frac{1}{2}mv^2 + \frac{1}{2}MV^2$



cons. of P $\Rightarrow$ O = MV – mv

cons. of L $\Rightarrow \left(\frac{1}{12}m\ell^2\right)w = O + MV\frac{\ell}{2}$

L around initial mid point of stick

Three egs, there unknown (v, V, M)

$E : \frac{1}{2}Iw^2 = \frac{p^2}{2m} + \frac{p^2}{2M}$

$L : Iw = P\frac{\ell}{2} \Rightarrow P = \frac{2Iw}{\ell}$

$\frac{1}{2}Iw^2 = \left(\frac{2Iw}{\ell}\right)^2\left(\frac{1}{2m} + \frac{1}{2M}\right)$

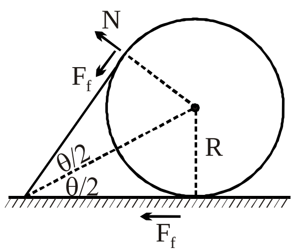
$\Rightarrow 1 = \frac{4I}{\ell^2}\left(\frac{1}{m} + \frac{1}{M}\right)$

$\Rightarrow I = \frac{4\left(\frac{1}{12}m\ell^2\right)}{\ell^2}\left(\frac{1}{m} + \frac{1}{M}\right)$

$\Rightarrow 3 = 1 + \dfrac{m}{M} \Rightarrow M = \dfrac{m}{2}$

Q.7 [5]

Let N be the normal force between the stick and the circle, and let F_f be the friction force between the ground andthe circle (see figure). Then we immediately see that the friction force between the stick and the circle is also F_f, because the torques from the two friction forces on the circle must cancel.



Looking at torques on the stick around the point of contact with the ground, we have Mg cos θ (L/2) = NL, where M is the mass of the stick and L is its length. Therefore, N = (Mg/2) cos θ. Balancing the horizontal forces on the circle then gives N sin θ = $F_f + F_f$ cos θ. So we have

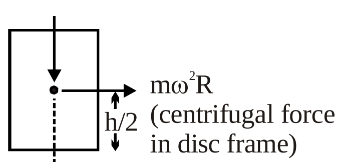
$$F_f = \frac{N\sin\theta}{1+\cos\theta} = \frac{Mg\sin\theta\cos\theta}{2(1+\cos\theta)}$$

But M = ρL, and from figure. we have L = R/tan (θ/2). Using the identity tan (θ/2) = sin θ/ (1 + cos θ), We finally obtain

$$F_f = \frac{1}{2}\rho g R \cos\theta.$$

Q.8 [25]

Cylinder will topple about right bottom



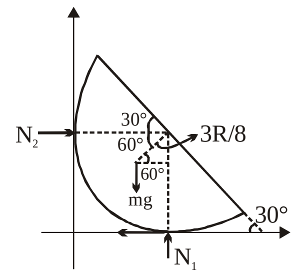
$$\frac{h}{2} mw^2R \geq \frac{D}{2} mg$$

$$w^2 \geq \frac{Dg}{Rh}$$

$$w_{min} = \sqrt{\frac{Dg}{Rh}}$$

Q.9 [6]

$$f = mg \times \frac{3R}{8} \cos 60°$$



N_2 = f

N_1 = mg

$$f = \frac{3mg}{16}$$

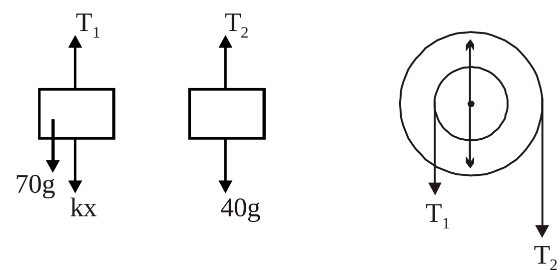
$f \leq \mu N_1$

$\mu \geq \dfrac{3}{16}$ $\Rightarrow 32\mu = 6$

Q.10 [84]

m_1 = 9 gm ; m_2 = 42 gm ; m_3 = 84 gm

Q.11 [100]



	$T_1 = 70g + kx$	...........(i)
	$T_2 = 40$ g	...........(ii)
	$T_1 (0.3) = T_2 (0.6)$	...........(iii)
$\Rightarrow$	70g + kx = 80g	
	kx = 10 g	$\therefore$ x = 1m

Q.12 [11]

$\tau = 19 \times \dfrac{20}{100} - 12 \times \dfrac{5}{100} =$ (3.8 – 0.6) N–m

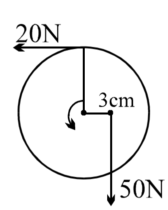
= 3.2 N–m (anticlockwise)

$\alpha = \dfrac{\tau}{I} = \dfrac{3.2}{32} = 0.1$ rad/sec^2 ; $\therefore \omega = \omega_0 + \alpha t$

$\Rightarrow$ 10 rad/sec + 0.1 ×10 rad/sec ; $\Rightarrow$ 11 rad/sec.

Q.13 [50]

$$20 \times 0.1 - 50 \times 0.03 = \frac{1}{2} mR^2\alpha$$



$2 - 1.50 = \frac{1}{2} \times 2 \times (0.1)2\ \alpha$

$0.5 = 0.01\ \alpha$

so rad / sec^2 (anticlockwise)

2nd Method

Taking outwords as (+ ive)

$\tau = +20 \times 30\text{N-cm} - 50 \times 3\ \text{N-cm}$

$= 50\ \text{N-cm} = 0.5\ \text{N-m}$

$$I = \frac{MR^2}{2}$$

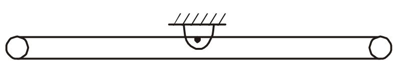
$$\tau = I\alpha \qquad \Rightarrow \alpha = \frac{\tau}{I} = \frac{0.5\ N-m \times 2}{2 \times (0.1)^2\ kgm^2}$$

$= 50\ rad/s^2$

Q.14 [158]

$$160 = 80 \times 1^2 + 60 \times 1^2 + \frac{M}{12} \times 2^2$$

$$\Rightarrow 200 = \frac{M}{3} \Rightarrow M = 600\ kg$$



$$x_c = \frac{0 \times 60 \times 60 \times -1 + 80 \times 1}{60 + 80 + 600} = \frac{20}{200} = 0.1m$$

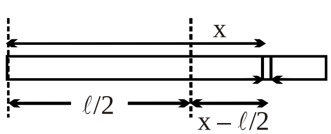
$I = I_c + Mx^2 \Rightarrow I_c = 160 - 200 \times (0.1)^2 = 158\ kg\ m^2$

Q.15 [55]

$$I_{sys} = \frac{mr^2}{2} + \left[\frac{mr^2}{2} + m(2r)^2\right] \times 6$$

$$= \frac{55}{2} mr^2 = 55$$

Q.16 [1]



$$dI = \int dm\left(x - \frac{\ell}{2}\right)^2$$

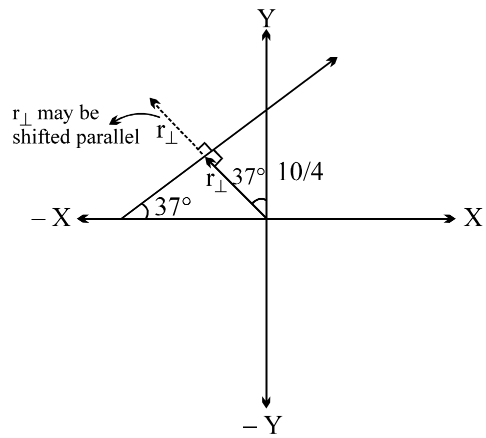
$$= r_0\ \rho_0 \int_{\ell/2}^{\ell/2} \left(1 + \frac{x}{\ell}\right)\left(1 - \frac{x}{\ell}\right)^2 dx$$

$$= \rho_0 \int_0^{\ell} \left(x^2 + \frac{\ell^2}{4} - \ell x + \frac{x^3}{\ell} + \frac{x\ell^2}{4} - x^2\right) dx$$

$$= \rho_0 \left[\frac{\ell^3}{4} - \frac{3\ell^3}{8} + \frac{\ell^3}{4}\right]$$

$$= \frac{\rho_0 \ell^3}{8} = 1\ kg\ m^2$$

Q.17 [120]



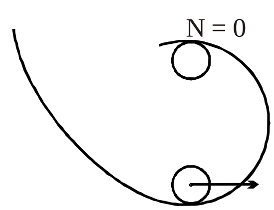
$$Y = \frac{3}{4}x + \frac{10}{4}, \quad Y = \tan 37° x + \frac{10}{4}$$

$r_{\perp} = 2m$

$L = mvr_{\perp} = 10 \times 6 \times 2 = 120\ kg\ m^2 / sec$

Q.18 [27]

$$mg = \frac{mv^2}{R}$$



$$\Rightarrow v = \sqrt{gR}$$

$$\frac{1}{2}mv^2 + \frac{1}{2} \times \frac{2}{5} mR^2 \omega^2 = mg(h - 2R)$$

$$\Rightarrow \frac{7}{10}v^2 = g(h - 2R)$$

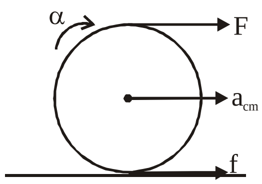
$$\frac{7}{10}gR = g(h - 2R)$$

$$h = \frac{27R}{10} = 2.7 \times 10$$

Q.19 [5]

F + f = ma

FR – fR = Ia/R

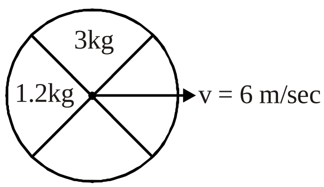


$\therefore\ F = \frac{5}{6}ma$

$\therefore\ a = \frac{6F}{5M}$

Q.20 [155]

$KE = \frac{1}{2}mv_{CM}^2 + \frac{1}{2}I_{CM}\omega^2$



$= \frac{1}{2}(7.8)\,(5)^2 + \frac{1}{2}(4.6)R^2\left(\frac{5}{R}\right)^2$

$= \frac{1}{2} \times 25 \times 12.9 = 155$ J

KVPY
PREVIOUS YEAR'S

Q.1 (C)

By using parallel axis theorem, $I = \frac{111}{2}mr^2$

Q.2 (A)

$\tau_{Net} = F\left(\frac{R}{2}\right) + FR + 2FR = 3.5FR$

Q.3 (C)

Apply conservation of linear momentum

mv = (m + M)v_0

$mv\sin\theta R = \left(\frac{2}{5}MR^2 + mR^2\right)\omega_0$

$mv\left(\frac{h-R}{R}\right)R = \frac{(2M+5M)}{5}\omega_0 R^2$

$(m+M)\,(h-R)\omega_0 R = \frac{(2M+5M)}{5}\omega_0 R^2$

$\frac{h}{R} = \frac{10m+7M}{5(m+M)}$

Q.4 (A)

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\frac{2}{5}mR^2\frac{v^2}{R^2}$

$mgh = \frac{7}{10}mv^2$

$v = \sqrt{\frac{10gh}{7}}$

Q.5 (A)

2 + S System lie above edge of 1.

$\frac{M}{2}y - M\left(\frac{L}{2} - y\right) = 0$

$\frac{y}{2} + y = \frac{L}{2}$

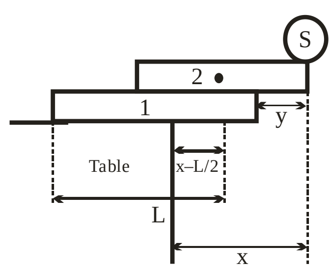
$y = \frac{L}{3}$

Now, 1 + 2 + S centre of mass will lie above the table

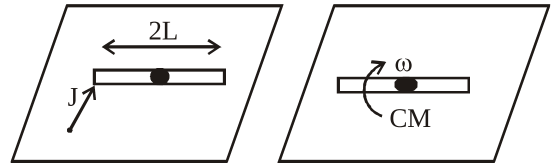
$\frac{3M}{2}\left(x - \frac{L}{3}\right) + M\left(x - \frac{L}{3} - \frac{L}{2}\right) = 0$

$\frac{3x}{2} - \frac{L}{2} + x - \frac{L}{3} - \frac{L}{2} = 0$

$\frac{5x}{2} = \frac{4L}{3} \Rightarrow x = \frac{8L}{15}$



Q.6 (C)



J = mv

....(i)

where v is the velocity of centre of mass.

After impulse rod get angular velocity ω
Angular impulse = Iω

$$J \times L = = \frac{m(2L)^2}{12} \times \omega$$

...(ii)

$$J = \frac{mL\omega}{3}$$

$$\boxed{\omega = \frac{3J}{mL}}$$

from equation (1); $v = \dfrac{J}{m}$

Kinetic energy = KE = $\dfrac{1}{2}mv^2 + \dfrac{1}{2}I\omega^2$

$$\Rightarrow \frac{1}{2}m\frac{J^2}{m^2} + \frac{1}{2} \times \frac{m \times 4L^2}{12} \times \frac{9J^2}{m^2J^2}$$

$$\Rightarrow \frac{J^2}{2m} + \frac{36J^2}{24m}$$

$$\Rightarrow \frac{48J^2}{24m} \Rightarrow \frac{2J^2}{m}$$

Q.7 (B)
If perfect rolling (solid cylinder P)
According to energy conservation law

$$mgh = \frac{1}{2}mv_P^2 + \frac{1}{2}I\left(\frac{V_P}{R}\right)^2$$

Here,
I → moment of intertia, R → Radius

$$I = \frac{mR^2}{2}$$

$$\omega = \frac{V_P}{R}$$

$$mgh = \frac{1}{2}mv_P^2 + \frac{1}{2}\frac{mR^2}{2}\frac{v_P^2}{R^2}$$

$$mgh = \frac{1}{2}mv_P^2\left[1 + \frac{1}{2}\right] = \frac{1}{2}mv_P^2 \times \frac{3}{2}$$

$$mgh = \frac{4}{3}mv_P^2$$

....(i)
If sliding without friction
(solid cylinder Q)
According to energy conservation law

$$mgh = \frac{1}{2}mv_Q^2$$

$$\Rightarrow v_Q^2 = 2gh$$

...(ii)
from equation (i) and (ii)

$$\frac{v_Q^2}{v_P^2} = \frac{2gh}{\left(\frac{4}{3}gh\right)} = \frac{3}{2}$$

$$\frac{v_Q}{v_P} = \sqrt{\frac{3}{2}}$$

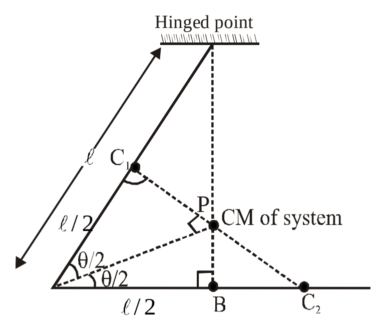
Q.8 (C)
Initial sphere is slipping and finally it start rolling During its motion τ about point of contact is zero.
∴ Angular momentum of sphere about point of contact remain conserved.

$Iw = (I + MR^2)\,\omega'$

$$\frac{2}{5}MR^2\omega = \left(\frac{2}{5}MR^2 + MR^2\right)\omega'$$

$$\omega' = \frac{2\omega}{7}$$

Q.9 (A)



When CM of system and Hinged point lie on one line then only system can remain in equilibrium in given position.

$$AB = \ell \cos\theta$$

$$AP = \frac{\ell}{2}\cos\frac{\theta}{2}$$

$$\cos\frac{\theta}{2} = \frac{AB}{AP} \Rightarrow AB = AP\cos\frac{\theta}{2}$$

$$\ell\cos\theta = \frac{\ell}{2}\cos^2\frac{\theta}{2}$$

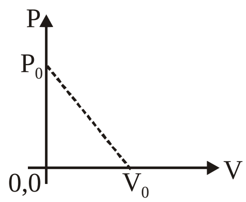
$$2\cos\theta = \frac{1+\cos\theta}{2}$$

$4\cos\theta = 1 + \cos\theta$

$3\cos\theta = 1$

$$\cos\theta = \frac{1}{3} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{3}\right)$$

Q.10 (C)



$$P = P_0 - \frac{P_0}{V_0} \times V$$

.....(1)

$PV = RT$

from (1) and (2)

$$\frac{RT}{V} = P_0 - \frac{P_0}{V_0} \times V$$

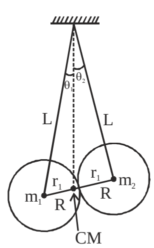
$$T = \frac{P_0 V}{R} - \frac{P_0 V^2}{RV_0} = \frac{P_0}{R}\left[V - \frac{V^2}{V_0}\right]$$

$$T = \frac{P_0 V}{R}\left[1 - \frac{V}{V_0}\right]$$

Q.11 (C)

Pole star is a visible star preferably a prominent one that is approximately aligned with the axis of rotation of earth.

Q.12 (B)



Using concept of COM

$m_1 r_1 = m_2 r_2$

$r_1 + r_2 = 2R$

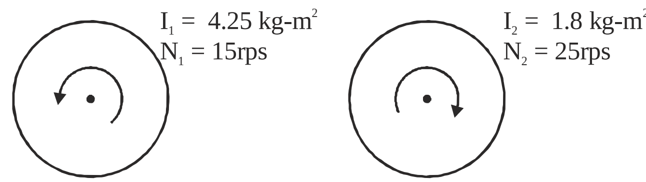
$$\left(\frac{m_2}{m_1} + 1\right) r_2 = 2R$$

$$r_2 = \frac{2m_1 R}{m_1 + m_2}$$

$L \sin\theta_2 = r_2$ [R < < L]

$$\theta_2 = \frac{2m_1 R}{(m_1 + m_2)L}$$

Q.13 (D)



Their axis of rotation is common.

Angular momentum conservation $I_1\omega_1 - \omega_2 I_2 = (I_1 + I_2)\omega$

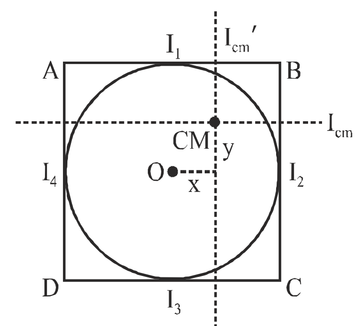
$2\pi(4.25)\,N_1 - 2\pi\,(1.8)\,N_2 = (4.25 + 1.80)\,N\,(2\pi)$

$(4.25 \times 15 - 1.8 \times 25) = (6.05)\,N$

$63.75 - 45 = 6.05\,N$

N = 3 rev/s.

Q.14 (A)



$I_{cm} + m(R + y)^2 = I_3$ ...(1)

$I_{cm} + m(R - y)^2 = I_1$ ...(2)

from (1) & (2)

$I_1 - I_3 = m[(R - y)^2 - (R + y)]$

$I_1 - I_3 = m(2R)(-2y)$ ...(3)

$I_{cm}' + m(R + x)^2 = I_4$ ...(4)

$I_{cm}' + m(R - x)^2 = I_2$ ...(5)

from (4) & (5)

$(I_2 - I_4) = m[(R - x)^2 - (R + x)^2]$

$I_2 - I_4 = m[(2R)(-2x)]$ ...(6)

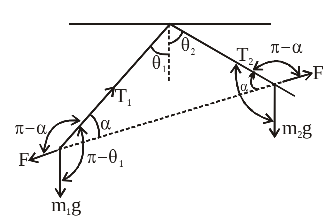
$(3)^2 + (6)^2$

$\Rightarrow (I_1 - I_3)^2 + (I_2 - I_4)^2 = (m^2 \times 4R^2 \times 4(x^2 + y^2)$

distance of CM from O = $\sqrt{x^2 + y^2}$

$$= \frac{1}{4mR}\sqrt{(I_1 - I_3)^2 + (I_2 - I_4)^2}$$

Q.15 (A)



$\theta_1 = 30^o, \theta_2 = 60^o$
using Lami theorem on m_1

$$\frac{F}{\sin(\pi-\theta_1)} = \frac{m_1 g}{\sin(\pi-\alpha)}$$

$$\frac{F}{\sin\theta_1} = \frac{m_1 g}{\sin\alpha} \quad ...(1)$$

using Lami theorem on m_2

$$\frac{F}{\sin(\pi-\theta_2)} = \frac{m_2 g}{\sin(\pi-\alpha)}$$

$$\frac{F}{\sin\theta_2} = \frac{m_2 g}{\sin\alpha} \quad ....(2)$$

using (1) & (2)
$m_1 \sin\theta_1 = m_2 \sin\theta_2$
$m_1 \times \sin 30^o = m_2 \sin 60^o$

$$\frac{m_1}{m_2} = \sqrt{3} = 1.7$$

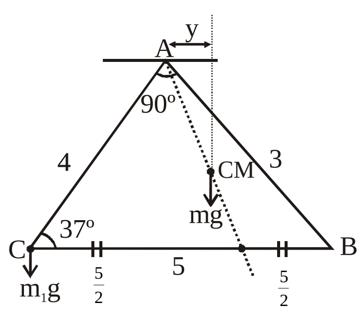
Q.16 (B)

$$\frac{1}{2}mV^2 + \frac{1}{2}mR^2\left(\frac{V}{R}\right)^2 = mgh \ \{\text{Using conservation of energy}\}$$

$$m\left(\frac{V^2}{2} + \frac{V^2}{2}\right) = mgh$$

$$h = \frac{V^2}{g}$$

Q.17 (C)



CM of triangular plate is on the median. If we put a mass say m_1 on C it will produce torque about A which balance the torque produce mg about A. Thus plate will can be in equilibrium position
$m_1 g \times 4 \cos 37 = mg \times y$

$$m_1 g \times 4 \times \frac{4}{5} = mg \times y$$

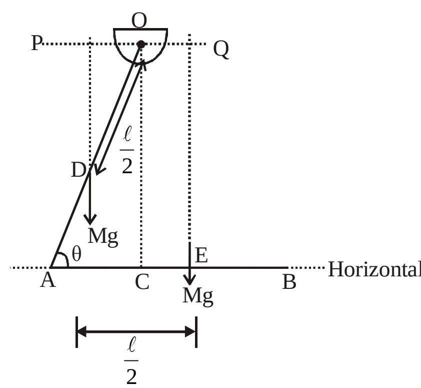
$$m_1 = m \times y \times \frac{5}{16}$$

$$\frac{m_1}{m} = y \times \frac{5}{16}$$

$y < 3$ $\quad\quad \therefore \frac{m_1}{m} < 1$

$m_1 < m$
$m_1 < 540$ g
from given option **Ans.** (A)

Q.18 (A)



For one arm to remain horizontal the net torque about O must be zero (in the position shown in the figure) for this OP = OQ

$$\Rightarrow OQ = \frac{\ell}{2}\cos\theta$$

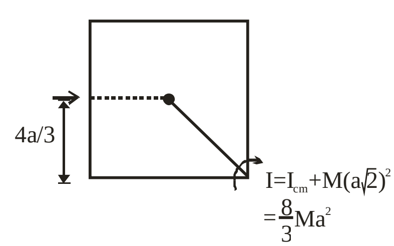
from figure
AE = AC + CE
$\Rightarrow AE = \ell\cos\theta + OQ$

$$= \frac{\ell}{2} = \ell\cos\theta + \frac{\ell}{2}\cos\theta$$

$$\Rightarrow \cos\theta = \frac{1}{3}$$

hence $\theta = \cos^{-1}(1/3)$
correct Answer is (A)

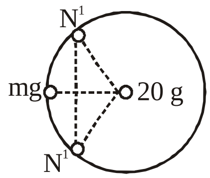
Q.19 (D)



$\therefore mvr = I\omega$

$$\Rightarrow \omega = \frac{mv\left(\frac{4a}{3}\right)}{\frac{8}{3}Ma^2} = \frac{mv}{2Ma}$$

Q.20 (C)



$$(M+20)g\frac{R}{\sqrt{2}} = mgR$$

$$M+20=\sqrt{2}\,m\ ;\ M=\frac{20}{\sqrt{2}-1}=48.3\,kg$$

Q.21 (B)

Q.22 (C)

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$I = \frac{2}{5}mR^2$$

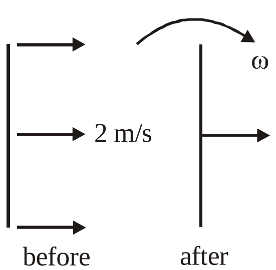
$v = R\omega$

$$\Rightarrow\ v=\sqrt{\frac{10gh}{7}} = 10\ m/s$$

On elastic collision with block velocity will interchange speed of block after collision is 10 m/s.

Q.23 (B)

By conservation of angular momentum



$L_i = L_f$

about feet on fixed ground

$$\frac{1.5}{2}\times m\times 2=\frac{m(1.5)^2}{3}\omega$$

ω = 2 rad/s

Q.24 **(B)**

The frictional force on the tyres is an external force and is being provided by the road.

Other options i.e. front tyre, rear tyre and brakes comprise the internal parts of bicycle thus forces applied by them will be internal only.

Q.25 **(D)**

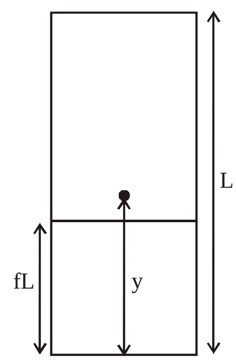
Mass of bottle = m0

Length of bottle = L

base Area = A = pr2

density of shampoo = r

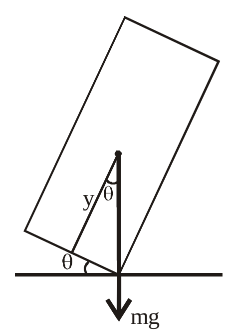
mass of shampoo = rfAL



Center of mass of system

$$y=\frac{m_0\frac{L}{2}+(\rho fAL)\left(\frac{fL}{2}\right)}{m_0+\rho fAL}$$

for critical angular displacement, mg will pass through tilted side.



From the diagram $\tan\theta = \frac{r}{y}$

$$\tan\theta=\frac{r(m_0+\rho ALf)}{\frac{L}{2}(m_0+\rho ALf^2)}$$

at f = 0 & f = 1, tipping angle 'θ' will be same. for very small values of 'f', we can neglect f^2 terms

$$\Rightarrow \tan\theta=\frac{r\ (m_0+\rho ALf)}{\frac{L}{2}\ m_0}$$

$$\theta = \tan^{-1}\left(\frac{r}{\frac{L}{2}}\frac{(m_0+\rho ALf)}{m_0}\right)$$

So if f increases θ will increase.

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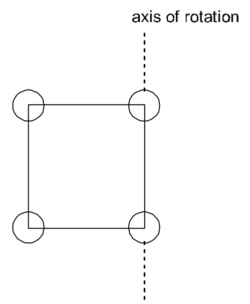
Q.1 (1)

$$Mg\,(\ell \sin\theta) = \frac{1}{2}MV_0^2 + \frac{1}{2}\times\frac{2}{5}MV_0^2$$

$$\therefore\ Mg\,(\ell\sin\theta) = MV^2\ \therefore\ \ell = \frac{7v^2}{10g\sin\theta}$$

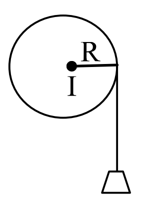
Q.2 (1)



$$I = 2\times\frac{2}{5}ma^2 + 2\left[\frac{2}{5}ma^2 + mb^2\right]$$

$$I = \frac{4}{5}ma^2 + \frac{4}{5}ma^2 + 2ma^2 = \frac{8}{5}ma^2 + mb^2$$

Q.3 (1)



mg – T = ma

TR = I α

a = Rα

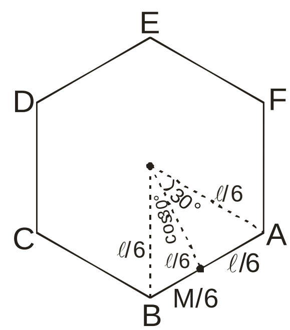
$$mg - \frac{I\alpha}{R} = ma$$

$$a = \frac{mg}{m + \frac{I}{R^2}}$$

$$V = \sqrt{\frac{2mgh}{m+\frac{I}{R^2}}} = \omega R$$

$$\omega^2 = \frac{2mgh}{I + mR^2}$$

Q.4 [0.8]



$$I_{AB} = \left[\frac{\frac{M}{6}\left(\frac{\ell}{6}\right)^2}{12} + \frac{M}{6}\left(\frac{\ell}{6}\frac{\sqrt{3}}{2}\right)^2\right]$$

$$I_{hexagon} = 6I_{AB} = M = \left[\frac{\ell^2}{12\times36} + \frac{\ell^2}{36}\times\frac{3}{4}\right]$$

$$= \frac{6}{100}\left[\frac{24\times24}{12\times36} + \frac{24\times24}{36}\times\frac{3}{4}\right]$$

$$= \frac{1}{100}[80] = 0.8\ kgm^2$$

Q.5 [8]

Ratio of time period

$$\frac{T_1}{T_2} = \frac{1}{8}$$

$$\frac{\frac{2\pi}{\omega_1}}{\frac{2\pi}{\omega_2}} = \frac{1}{8}$$

$$\frac{\omega_1}{\omega_2} = 8$$

Q.6 (1)

$I_1 = 2\frac{MR^2}{2}$

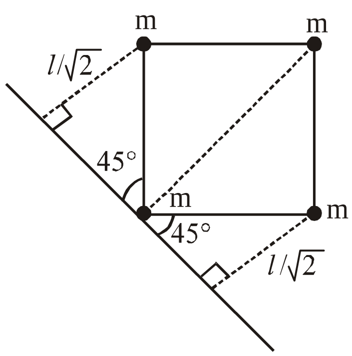
$I_2 = \frac{MR^2}{2}$

$I_3 = \frac{MR^2}{2}$

$I_4 = \frac{2}{5}MR^2$

Q.7 **(3)**

Moment of inertia of point mass
= mass × (Perpendicular distance from axis)



Moment of Inertia

$= m(0)^2 + m(l\sqrt{2})^2 + m\left(\frac{1}{\sqrt{2}}\right)^2 + m\left(\frac{1}{\sqrt{2}}\right)^2$

$= 3ml^2$

Q.8 **[20]**

$$\alpha = \frac{\tau}{I} = \frac{F.R.}{mR^2/2} = \frac{2F}{mR}$$

$$\alpha = \frac{2\times 200}{20\times(0.2)} = 10 rad/s^2$$

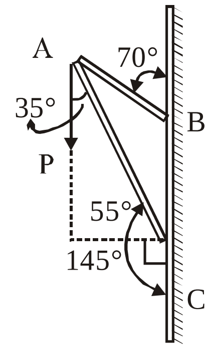
$$\omega^2 = \omega_0{}^2 + 2\alpha\Delta\theta$$

$(50)^2 = 0^2 + (10)\,\Delta\theta \Rightarrow \Delta\theta = \frac{2500}{20}$

$\Delta\theta = 125$ rad

No. of revolution = $\frac{125}{2\sum} \approx$ 20 revolution

Q.9 **[82]**



Component along AC
= 100 cos 35°N
= 100 × 0.819 N
= 81.9 N ≈ 82 N

Q.10 **[20]**

$\vec{\tau} = \vec{r}\times\vec{F}$

$\vec{r} = (2\hat{i}) - (2\hat{i} + 3\hat{j} + 4\hat{k}) = -3\hat{j} - 4\hat{k}$

& $\vec{F} = 4\hat{i} + 3\hat{j} + 4\hat{k}$

$$\vec{\tau} = \vec{r}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -3 & -4 \\ 4 & 3 & 4 \end{vmatrix}$$

$= \hat{i}(-12+12) - \hat{j}(0+16) + \hat{k}(0+12)$

$= -16\hat{i} + 12\hat{k}$

$\therefore\ |\vec{\tau}| = \sqrt{16^2 + 12^2} = 20$

Q.11 **[3]**

$$a = \frac{g\sin\theta}{1+\frac{I}{mR^2}} = \frac{g\sin\theta}{1+\frac{1}{2}} = \frac{2}{3}g\sin\theta$$

b = 3

Q.12 **(4)**

We know, $\vec{L} = m(\vec{r}\times\vec{v})$

with respect to A, we always get direction of $\vec{L}$ along +ve z-axis and also constant magnitude as mvr. But with respect to B, we get constant magnitude but continuously changing direction.

Q.13 **[728]**

We know, $\theta = \left(\frac{\omega_1 + \omega_2}{2}\right)t$

Let number of revolutions be N

$\therefore\ 2\pi N = 2\pi\left(\frac{900+2460}{60\times 2}\right)\times 26$

N = 728

Q.14 [4]

Mg sinθ R = (mk$_2$ + mR$_2$) α

$$\alpha = \frac{Rg\sin\theta}{k^2+R^2} \Rightarrow a = \frac{g\sin\theta}{1+\frac{k^2}{R^2}}$$

$$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2s}{g\sin\theta}\left(1+\frac{k^2}{R^2}\right)}$$

for least time, k should be least & we know k is least for solid sphere.

Q.15 **(3)**

$$(3)\ a = \frac{g\sin\theta}{1+\frac{I}{mR^2}} = \frac{5}{7}\times\frac{10}{2} = \frac{25}{7}$$

$$t = \frac{2v_0}{a} = \frac{2\times1\times7}{25} = 0.56$$

Q.16 **(3)**

Using conservation of angular momentum

(Mr2)ω = (Mr2 + 2mr2)ω'

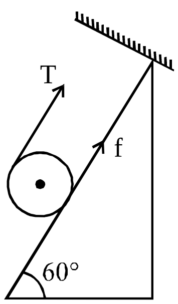
$$\omega = \frac{M\omega}{M+2m}$$

Q.17 **(3)**

$$\pi = \Rightarrow = \frac{L}{\pi}$$

$$I = Mr^2 = \frac{ML^2}{\pi^2}$$

Q.18 **(3)**



Let's take solid cylinder is in equilibrium

T + f = mg sin60 ....(i)

TR – fR = 0 ....(ii)

Solving we get

$$T = f_{req} = \frac{mg\sin\theta}{2}$$

But limiting friction < required friction

$$\mu mg\cos 60° < \frac{mg\sin 60°}{2}$$

∴ Hence cylinder will not remain in equilibrium

Hence f = kinetic

$= \mu_k N$

$= \mu_k mg\cos 60°$

$= \frac{mg}{5}$

Q.19 **[200]**

Q.20 [3]

Q.21 **(2)**

Q.22 **[2]**

Q.23 **(1)**

Q.24 (2)

Q.25 [4]

Q.26 (4)

Q.27 (3)

Q.28 [9]

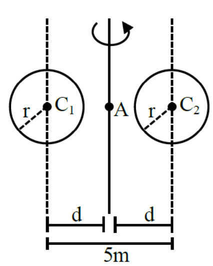
Q.29 (3)

Q.30 (4)

Q.31 [52]

Q.32 (2)

Q.33 (3)

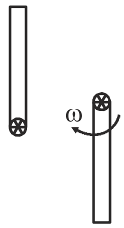


M = 1.5 kg, r = 0.5 m, $d = \frac{5}{2}$ m

$$I = 2\left(\frac{2}{5}Mr^2 + Md^2\right) = 19.05\ kgm^2$$

Q.34 (4)

Q.35 [6]



by energy conservation $mg\ell = \frac{1}{2}I\omega^2 = \frac{1}{2}\frac{m\ell^2\omega^2}{3}$

$\Rightarrow \omega = \sqrt{\frac{6g}{\ell}}$

Speed $v = \omega r = \omega\ell = \sqrt{6g\ell}$

$v = \sqrt{6\times10\times.6} = 6m/s$

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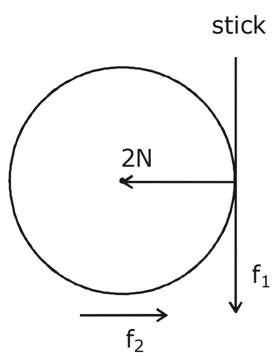
Q.1 **[0004]**

$2 - f_2 = 2a = 0.6 \Rightarrow f_2 = 1.4$

$\tau = I\alpha \Rightarrow (f_2 - f_1)R = MR^2\frac{a}{R}$

$1.4 - f_1 = Ma = 0.6$



$f_1 = 0.8 = \mu(2) = \frac{P}{10}\times2$

$P = 4$

Q.2 **(B)**

$\tau = \frac{dL}{dt} = \frac{d}{dt}(I\omega)$

$\tau = \omega\frac{dI}{dt} = \omega\frac{d}{dt}(I_{rod} + I_m)$

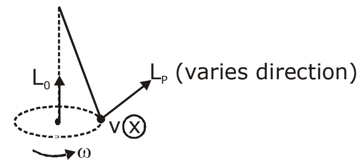
as I_{rod} = com $\Rightarrow \tau = \frac{wd}{dt}\left(I_{insect}\right)$

$= \omega\frac{d}{dt}(mr^2) = m\omega\left(2r\frac{dr}{dt}\right) = 2m\,r\omega v$

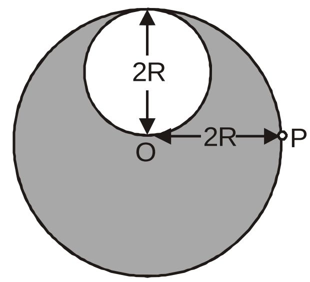
$= 2m(vt)\omega v \Rightarrow \tau \propto t$

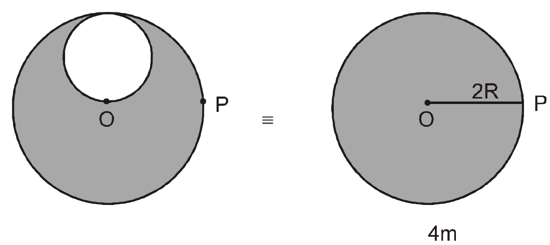
Q.3 **(C)**

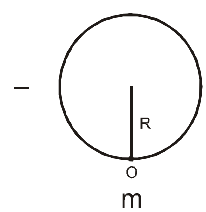
L_0 remains cons. in magnitude and direction but L_P changes its direction continously hence L_P is variable



Q.4 **[3]**

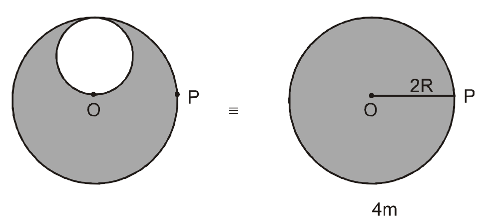


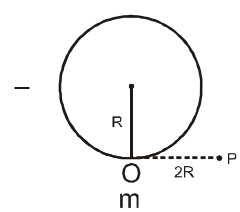




$I_0 = \frac{(4m)(2R)^2}{2} - \frac{3}{2}mR^2 = mR^2\left[8 - \frac{3}{2}\right]$

$= \frac{13}{2}mR^2$





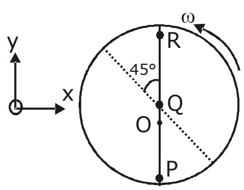
$$I_P = \frac{3}{2}\ (4m)\ (2R)^2 - \left[\frac{mR^2}{2} + m[(2R)^2 + R^2]\right]$$

$$= 24\ mR^2 - \frac{11}{2}mR^2$$

$$= \frac{37}{2}mR^2$$

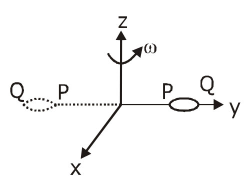
$$\frac{I_P}{I_O} = \frac{\frac{37}{2}}{\frac{13}{2}} = \frac{37}{13} \approx 3$$

Q.5 **(C)**

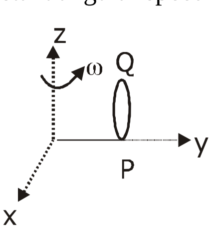


At 45° P & Q both land in unshaded region.

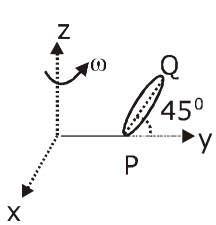
The general motion of a rigid body can be considered to be a combination of (i) a motion of its centre of mass about an axis, and (ii) its motion about an instantaneous axis passing through the centre of mass. These axes need not be stationary. Consider, for example, a thin uniform disc welded (rigidly fixed) horizontally at its rim to a massless stick, as shown in the figure. When the disc-stick system is rotated about the origin on a horizontal frictionless plane with angular speed ω, the motion at any instant can be taken as a combination of (i) a rotation of the centre of mass of the disc about the z-axis, and (ii) a rotation of the disc through an instantaneous vertical axis passing through its centre of mass (as is seen from the changed orientation of points P and Q). Both these motions have the same angular speed ω in this case.



Now consider two similar systems as shown in the figure: Case (a) the disc with its face vertical and parallel to x-z plane; case (b) the disc with its face making an angle of 45° with x-y plane and its horizontal diameter parallel to x-axis. In both the cases, the disc is welded at point P, and the systems are rotated with constant angular speed ω about the z-axis.



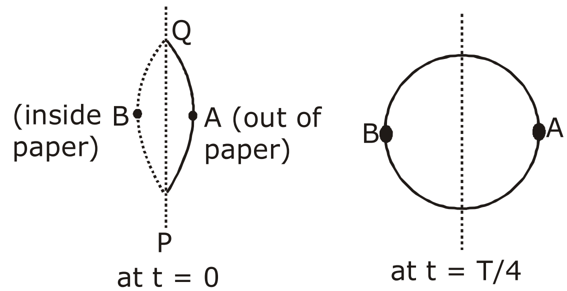
Case (a)



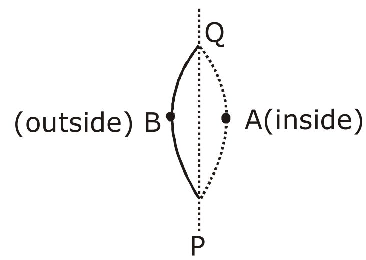
Case (b)

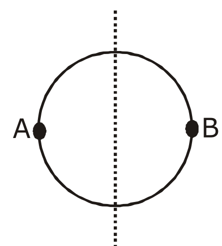
Q.6 **(A)**

Consider case (a)



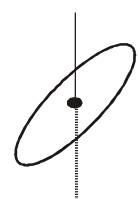
at t = T/2 at t = 3T/4





Hence axis is vertical.

For case (b)

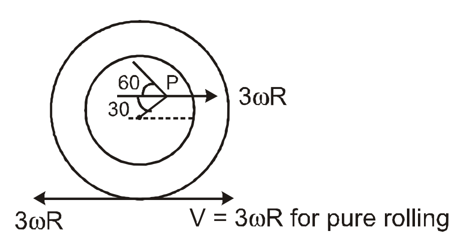


Q.7 **(D)**

Angular Velocity of rigid body about any axes which are parallel to each other is same . So angular velocity is ω .

Q.8 **(A,B)**

$V_0 = 3\omega R\,\hat{i}$



$V_P\ (3\omega R - \frac{\omega R}{2}\cos 60^\circ)\ \hat{i} + \frac{\omega R}{2}\sin 60\,\hat{j}$

$= \frac{11\omega R}{4}\hat{i} + \frac{\sqrt{3}\omega R}{4}\hat{i}$

Q.9 **(D)**

$I_P > I_Q$

$a = \frac{g\sin\theta}{1 + I/MR^2}$

Hence $a_p < a_0$

$t_p > t_Q$

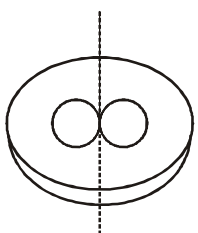
$V_p < v_Q$

And as $\omega = v/R$

So $\omega_P < \omega_Q$

Q.10 **[8]**

Angular momentum conservation



$I_1\omega_1 = I_2\omega_2$

$\frac{MR^2}{2}\omega_1 = \left[MR^2 + 2(mr^2 + mr^2)\right]\omega_2$

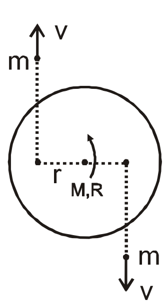
$= \frac{50(0.4)^2}{2} \times 10$

$= \left[\frac{50(0.4)^2}{2} + 2\{(6.25)(0.2^2 + 0.2^2)\}\right]\omega_2$

$40 = [4 + 1]\,\omega_2 \Rightarrow \quad \omega_2 = 8$ rad/s

Q.11 **[4]**

Applying conservation of angular momentum.



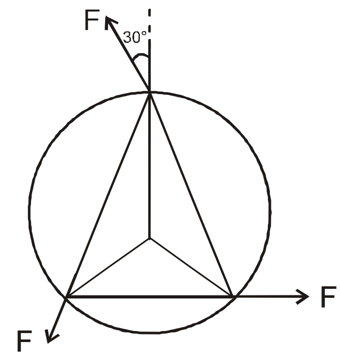
$2mvr - \frac{MR^2}{2}\omega = 0$

$\omega = \frac{4mvr}{MR^2}$

$\omega = \frac{(4)(5\times10^{-2})(9)\left(\frac{1}{4}\right)}{45\times10^{-2}\times\frac{1}{4}}$

$\omega = 4$ rad/s

Q.12 **[2]**



$\omega = \frac{\int \tau dt}{I} = \frac{\int_0^t 3F\sin 30^\circ R\,dt}{I}$

$= \frac{3.(0.5)\ (0.5)\ (0.5)\ (1)}{\frac{1.5(0.5)^2}{2}} \quad = 2$ rad/s

Q.13 [7]

Q.14 **(D)**

Q.15 **[6]**

Q.16 **(D)**

At equilibrium, reaction of the wall on the stick cannot be equal in magnitude to the reaction of the floor on the stick.

Q.17 (A, B, D)

$\vec{r}(t) = \propto t^3\hat{i} + \beta t^2\hat{j}$

$\vec{v} = \frac{d\vec{r}}{dt} = 3\propto t^2\hat{i} + 2\beta t\,\hat{j}$

$\Rightarrow (\vec{r})_{t=1} = \frac{10}{3}\hat{i} + 5\hat{j}$

$(\vec{v})_{t=1} = 10\hat{i} + 10\hat{j}$

$\left(\vec{p}\right)_{t=1} = \hat{i} + \hat{j}$

$\vec{L} = \vec{r} \times \vec{p} = \left(\frac{5}{3}\right)\hat{k}$

$\vec{F} = m\frac{d\vec{v}}{dt}$

$\left(\vec{F}\right)_{t=1} = 2\hat{i} + \hat{j}$

$\vec{\tau} = \vec{r} \times \vec{F} = -\left(\frac{20}{3}\right)\hat{k}$

Hence, (a, b, d)

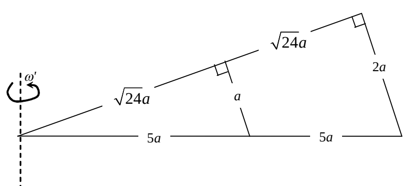
Q.18 (A,D)

As the discs are rolling without slipping

$\therefore \omega' \times 5a = \omega a \qquad \Rightarrow \omega' = \frac{\omega}{5}$

Angular momentum of system about CM through an axis along rod is

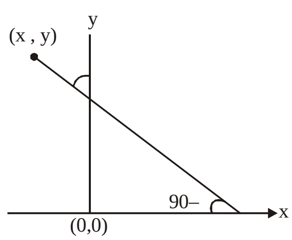
$= \frac{ma^2}{2} + \frac{4m(2a)^2}{2} = \frac{17ma^2}{2}$



Hence, (B) or (a, b)

Q.19 **(A, or AB)**

Q.20 **(BCD)**



$x = -\frac{\ell}{2}\sin\theta$

y = $\ell\cos\theta$

$\frac{y^2}{\ell^2} + \frac{4x^2}{\ell^2} = 1$

Path of A is ellipse

(B) torque about point of contact

$mg\frac{\ell}{2}\sin\theta = I\alpha$

hence torque $\propto$ sin θ

(C) $y_{cm} = \frac{L}{2}(1 - \cos\theta)$

(D) midpoint will fall vertically downwards

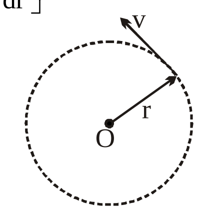
Q.21 **(Bonus)**

Q.22 **(A)**

Q.23 (B,C)

$v = \frac{kr^2}{2}$

F = – kr (towards centre) $\left[F = \frac{dv}{dr}\right]$



At r = R,

$kR = \frac{mv^2}{R}$ [Centripetal force]

$v = \sqrt{\frac{kR^2}{m}} = \sqrt{\frac{k}{m}}R$

$L = m\sqrt{\frac{k}{m}}R^2$

Q.24 (A,C)

$\vec{F} = (\alpha t\hat{i} + \beta\hat{j})$ [At t = 0, v = 0, $\vec{\tau} = \vec{0}$]

α = 1, β = 1

$\vec{F} = t\hat{i} + \hat{j}$

$m\frac{d\vec{v}}{dt} = t\hat{i} + \hat{j}$

On integrating

$m\vec{v} = \frac{t^2}{2}(\hat{i} + t\hat{j})$ [m = 1kg]

$\frac{d\vec{r}}{dt} = \frac{t^2}{2}\hat{i} + t\hat{j}$ $[\vec{r} = \vec{0}$ at $t = 0]$

On integrating

At t = 1 sec, $\vec{\tau} = (\vec{r} \times \vec{F}) = \left(\frac{1}{6}\hat{i} + \frac{1}{2}\hat{j}\right) \times (\hat{i} + \hat{j})$

$\vec{\tau} = -\frac{1}{3}\hat{k}$

$\vec{v} = \frac{t^2}{2}\hat{i} + t\hat{j}$

At = t = 1 $\left(\frac{1}{2}\hat{i} + \hat{j}\right) = \frac{1}{2}\left(\hat{i} + 2\hat{j}\ m/sec\right)$

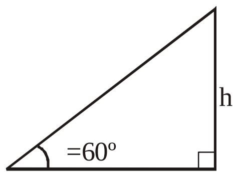
At t = 1 $\vec{s} = \vec{r}_1 - \vec{r}_0$

$$=\left[\frac{1}{6}\hat{i}+\frac{1}{2}\hat{j}\right] = [\vec{0}]$$

$$\vec{s}=\frac{1}{6}\hat{i}+\frac{1}{2}\hat{j}$$

$$|\vec{s}|=\sqrt{\left(\frac{1}{6}\right)^2+\left(\frac{1}{2}\right)^2} \Rightarrow \frac{\sqrt{10}}{6}\ \text{m}$$

Q.25 [0.75 m]



$$a_c=\frac{g\sin\theta}{1+\frac{I_C}{MR^2}}$$

$$a_{ring}=\frac{g\sin\theta}{2}$$

$$a_{disc}=\frac{2g\sin\theta}{3}$$

$$\frac{h}{\sin\theta}=\frac{1}{2}\left(\frac{g\sin\theta}{2}\right)t_1^2 \Rightarrow t_1\sqrt{\frac{4h}{g\sin^2\theta}}=\sqrt{\frac{16h}{3g}}$$

$$\frac{h}{\sin\theta}=\frac{1}{2}\left(\frac{2g\sin\theta}{3}\right)t_2^2 \Rightarrow t_2=\sqrt{\frac{3h}{g\sin^2\theta}}=\sqrt{\frac{4h}{g}}$$

$$\Rightarrow \sqrt{\frac{16h}{3g}}-\sqrt{\frac{4h}{g}}=\frac{2-\sqrt{3}}{\sqrt{10}}$$

$$\sqrt{h}\left[\frac{4}{\sqrt{3}}-2\right]=2-\sqrt{3}$$

$$\sqrt{h}=\frac{(2-\sqrt{3})\sqrt{3}}{(4-2\sqrt{3})}=\frac{\sqrt{3}}{2}\Rightarrow h=\frac{3}{4}=0.75\text{m}$$

Q.26 (A)

(P) $\vec{r}(t)=\alpha t\hat{i}+\beta t\hat{j}$

$$\vec{v}=\frac{d\vec{r}(t)}{dt}=\alpha\hat{i}+\beta\hat{j}\ \{\text{constant}\}$$

$$\vec{a}=\frac{\overrightarrow{dv}}{dt}=0$$

$\vec{P}=m\vec{v}$ (remain constant)

$k=\frac{1}{2}mv^2$ (remain constant)

$$\vec{F}=\left[\frac{\partial U}{\partial x}\hat{i}+\frac{\partial U}{\partial y}\hat{i}\right]=0$$

$\Rightarrow U \rightarrow$ constant

E = K + U

$$\frac{d\vec{L}}{dt}=\vec{\tau}=\vec{r}\times\vec{F}=0$$

$\vec{L}=$ constant

(Q) $\vec{r}=\alpha\cos(\omega t)\hat{i}+\beta\sin(\omega t)\hat{j}$

$$\vec{v}=\frac{d\vec{r}}{dt}=-\alpha\omega\sin(\omega t)\hat{i}+\beta\omega\cos(\omega t)\hat{j}$$

$$\vec{a}=\frac{d\vec{v}}{dt}=-\alpha\omega^2\cos(\omega t)\hat{i}-\beta\omega^2\sin(\omega t)\hat{j}$$

$$=-\omega^2\left[\alpha\cos(\omega t)\hat{i}+\beta\sin(\omega t)\hat{j}\right]$$

$$a=-\omega^2\vec{r}$$

$\vec{\tau}=\vec{r}\times\vec{F}=0$ { $\vec{r}$ and $\vec{F}$ are parallel}

$$\Delta U=-\int\vec{F}.dr=+\int_0^r m\omega^2.r.dr$$

$$\Delta U=m\omega^2\left(\frac{r^2}{2}\right)$$

$$U\propto r^2$$

$$r=\sqrt{\alpha^2\cos^2(\omega t)+\beta^2\sin^2(\omega t)}$$

r is a function of time (t)

U depends on r hence it will change with time

Total energy remain constant because force is central.

(R) $\vec{r}(t)=\alpha\left(\cos\omega t\hat{i}+\sin(\omega t)\hat{j}\right)$

$$\vec{v}(t)=\frac{d\vec{r}(t)}{dt}=\alpha\left[-\omega\sin(\omega t)\hat{i}+\omega\cos(\omega t)\hat{j}\right]$$

$|\vec{v}|=\alpha\omega$ (Speed remains constant)

$$\vec{a}=(t)=\frac{d\vec{v}(t)}{dt}=\alpha\left[-\omega^2\cos(\omega t)\hat{i}-\omega^2\sin(\omega t)\hat{j}\right]$$

$$=-\alpha\omega^2\left[\cos(\omega t)\hat{i}+\sin(\omega t)\hat{j}\right]$$

$$\vec{\alpha}(t)=-\omega^2(\vec{r})$$

$$\vec{\tau}=\vec{F}\times\vec{r}=0$$

$|\vec{r}|=\alpha$ (remain constant)

Force is central in nature and distance from fixed point is constant.

Potential energy remains constant

Kinetic energy is also constant (speed is constant)

(S) $\vec{r} = \alpha t\hat{i} + \frac{\beta}{2}t^2\hat{j}$

$\vec{v} = \frac{d\vec{r}}{dt} = \alpha t\,\hat{i} + \beta t\,\hat{j}$ (speed of particle depends on 't')

$\vec{a} = \frac{d\vec{v}}{dt} = \beta\hat{j}$ {constant}

$\vec{F} = m\vec{a} =$ {constant}

$$\Delta U = -\int \vec{F}.d\vec{r} = -m\int_0^t \beta\hat{j}.\left(\alpha\hat{i} + \beta t\hat{j}\right)dt$$

$$U = \frac{-m\beta^2 t^2}{2}$$

$$k = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\alpha^2 + \beta^2 t^2\right)$$

$E = k + U = \frac{1}{2}m\alpha^2$ (remain constant)

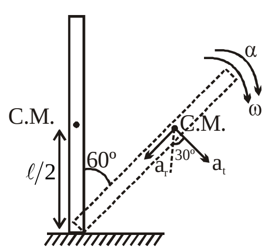
Q.27 (A,C,D)

We can treat contact point as hinged.
Applying work energy theorem
$W_g = \Delta K.E.$

$$mg\frac{\ell}{4} = \frac{1}{2}\left(\frac{m\ell^2}{3}\right)\omega^2$$

$$\omega = \sqrt{\frac{3g}{2\ell}}$$

Radial acceleration of C.M. of rod $= \left(\frac{\ell}{2}\right)\omega^2 = \frac{3g}{4}$



Using τ = I α about contact point

$$\frac{mg\ell}{2}\sin 60^\circ = \frac{m\ell^2}{3}\alpha$$

$$\Rightarrow \alpha = \frac{3\sqrt{3}}{4\ell}g$$

Net vertical acceleration of C.M. of rod
$a_v = a_r \cos 60^\circ + a_t \cos 30^\circ$

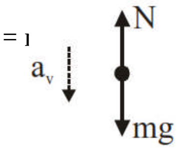
$$= \left(\frac{3g}{4}\right)\left(\frac{1}{2}\right) + \left(\alpha\frac{\ell}{2}\right)\cos 30^\circ$$

$$= \frac{3g}{4} + \frac{3\sqrt{3}}{4\ell}\left(\frac{\ell}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{3g}{8} + \frac{9}{16} = \frac{15}{16}g$$

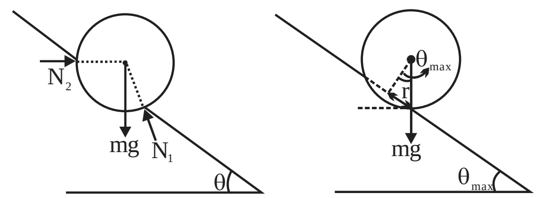
Applying F_{net} = ma in vertical direction on rod as system

`mg – N = ma_v =



$$\Rightarrow N = \frac{mg}{16}$$

Q.28 (A)



For θ_{max}, the football is about to roll, then N_2=0 and all forces (Mg and N_1) must pass through contact point

$$\therefore \cos(90^\circ - \theta_{max}) = \frac{r}{R} \Rightarrow \sin\theta_{max} = \frac{r}{R}$$

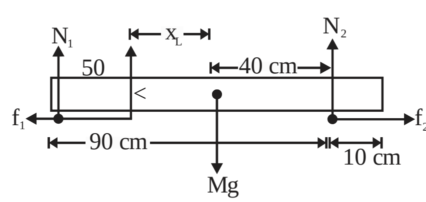
Q.29 (B)

For no slipping at the ground,
$V_{centre} = \omega R$ (R is radius of roller)
∴ Velocity of scale = (V_{center}+ωr) [r is radius of axle]
Given, $V_{center} \cdot t$= 50 cm
∴ Distance moved by scale = (V_{center}+ωr)t

$$= \left(V_{center} + \frac{V_{center}r}{R}\right)t = \frac{3V_{center}}{2}\cdot t = 75\text{cm}$$

Therefore relative displacement (with respect to centre of roller) is (75 – 50) cm = 25 cm

Q.30 [25.60]



Initially

$N_1 + N_2 = Mg$ $\qquad N_1 = \frac{4Mg}{9}$

$(\tau_N = 0)_{about centre} N_1(50)$ $\qquad N_2 = \frac{5Mg}{9}$

$5N_1 = 4N_2$

$f_{1_K} = \mu_K N_1$ $\qquad$ $f_{1_L} = \mu_S N_1$

$$\frac{f_{1_K} = 0.32N_1}{f_{2_K} = 0.32N_2} \qquad \frac{f_{1_L} = 0.4N_1}{f_{2_L} = 0.4N_2}$$

Suppose x_L=distance of left finger from centre when right finger starts moving

$(\tau_n=0)_{\text{about centre}} \Rightarrow N_1x_L=N_2(40)$

$f_{K_1} = f_{L_2} \Rightarrow 0.32N_1 = 0.40N_2$

$4N_1=5N_2$

$$N_1x_L = \frac{4N_1}{5}(40)$$

$x_L=32$

Now X_R=distance when right finger stops and left finger starts moving

$(\tau_n=0)$about centre $\Rightarrow N_1x_L=N_2(x_R)$

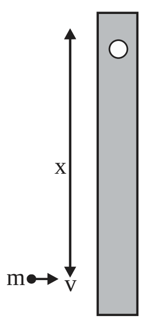
$f_{L_1} = f_{K_2} \Rightarrow 0.4N_1 = 0.32N_2$

$5N_1=4N_2$

$$\frac{4N_2}{5}(32) = N_2x_R$$

$$x_R = \frac{128}{5} = 25.6\,\text{cm}$$

Q.31 (A, C, D)



by the angular momentum conservation about the suspension point.

$$mvx = \left(\frac{m\ell^2}{3} + mx^2\right)\omega$$

$$\therefore \omega = \frac{mvx}{\frac{m\ell^2}{3} + mx^2} = \frac{2vx}{\ell^2 + 3x}$$

For maximum $\omega \Rightarrow \frac{d\omega}{dx} = 0$

$$\therefore x_M = \frac{\ell}{\sqrt{3}}$$

So the $\omega = \frac{V}{2\ell}\sqrt{3}$

Q.32 (BD)

Q.33 (ABC)

Q.34 [49]

Q.35 (ABD)

Q.36 [0.18]

Q.37 [0.16]